📚 Manzil 2026 Basic Maths for JEE

Brief Overview

This note covering **Basic Maths** was created from the Manzil 2026: BASIC MATHS in One Shot: All Concepts & PYQs Covered | JEE Main & Advanced YouTube video. It captures the full 358-minute lecture with in-depth explanations and practice problems tailored for JEE Main and Advanced aspirants.

Key Points

- Detailed coverage of **dimensional analysis** and unit conversions.
- Clear distinctions between vector and scalar physical quantities.
- Practical examples of force, momentum, and energy calculations.
- Insightful strategies for tackling JEE-style error-propagation questions.

📚 Course Overview & Study Resources

- **Platform**: *Pi App* – dedicated section for the "Mansil Series".
- **Features**:
    - Full-screen video (no YouTube distractions).
    - Class notes, DPPs, test series, live P&Y Q sessions.
    - **Ask-AI**: sketch a doubt → AI provides an instant solution.
- **Target Audience**: JEE Main first-attempt takers aiming for the 99-percentile, plus 11th/12th students and those needing revision.

🧭 Physical Quantities Classification

> *Physical quantity*: a measurable property of a system (e.g., length, mass, time).

- **Based on direction**
    - **Scalar** – magnitude only (e.g., *length*, *mass*, *temperature*).
    - **Vector** – magnitude + direction (e.g., *force*, *velocity*, *momentum*).
- **Based on dependence**
    - **Fundamental (base) quantities** – independent, not derived from others.
    - **Derived quantities** – obtained from fundamental ones (e.g., *density*, *acceleration*).

🔧 Seven Fundamental Physical Quantities & SI Units

Quantity	Symbol	SI Unit	Unit Symbol
Length	L	metre	m
Mass	M	kilogram	kg
Time	T	second	s
Temperature	Θ	kelvin	K
Electric current	I	ampere	A
Luminous intensity	J	candela	cd
Amount of substance	N	mole	mol

- These seven form the basis for all other (derived) quantities.

📐 Unit Systems

System	Length	Mass	Time	Typical Use
FPS (Foot-Pound-Second)	foot (ft)	pound (lb)	second (s)	Older engineering, US customary
CGS (Centimetre-Gram-Second)	centimetre (cm)	gram (g)	second (s)	Classical physics, early textbooks
MKS (Meter-Kilogram-Second)	metre (m)	kilogram (kg)	second (s)	Pre-SI, still common
SI (International System)	metre (m)	kilogram (kg)	second (s)	Modern scientific work; adds derived units (N, J, W, etc.)

- **Derived SI units**:
  - Force → newton (N) = $\mathrm{kg\cdotpm\cdotps^{-2}}$
  - Energy → joule (J) = $\mathrm{kg\cdotpm^2s^{-2}}$
  - Power → watt (W) = $\mathrm{kg\cdotpm^2s^{-3}}$

🔄 Unit-Conversion Practice

1. **Density conversion**
   - Given: $6\ \mathrm{g/cm^3}$.
   - Convert to $\mathrm{kg/m^3}$:
     - $1\ \mathrm{g} = 10^{-3}\ \mathrm{kg}$, $1\ \mathrm{cm} = 10^{-2}\ \mathrm{m}$.
     - $6\ \dfrac{\mathrm{g}}{\mathrm{cm}^3} = 6\ \dfrac{10^{-3}\ \mathrm{kg}}{(10^{-2}\ \mathrm{m})^3} = 6 \times 10^3\ \dfrac{\mathrm{kg}}{\mathrm{m}^3}$.
2. **Momentum example**
   - $p = 20\ \mathrm{kg}\cdotp\mathrm{m/s}$ (given in CGS as $20\ \mathrm{g}\cdotp\mathrm{cm/s}$).
   - Convert mass: $20\ \mathrm{g} = 0.02\ \mathrm{kg}$; length: $1\ \mathrm{cm} = 10^{-2}\ \mathrm{m}$.
   - Result: $p = 0.02\ \mathrm{kg} \times 0.01\ \mathrm{m/s} = 2 \times 10^{-4}\ \mathrm{kg}\cdotp\mathrm{m/s}$.
3. **General rule**
   - Keep the **exponent** for each base dimension (M, L, T) unchanged; only adjust numeric factor using powers of ten.

📏 Dimensional Analysis

> *Dimensional formula* expresses a quantity as a product of fundamental dimensions raised to powers.

- **Notation**: $[Q] = M^a L^b T^c \ldots$

Example: Density

- Definition: $\rho = \dfrac{\text{mass}}{\text{volume}}$
- Dimensional formula: $[\rho] = M^1 L^{-3}$ (mass M, length L cubed for volume).

Example: Force

- Definition: $F = ma$ (mass × acceleration).
- Acceleration: $a = \frac{L}{T^2}$
- Dimensional formula: $[F] = M^1 L^1 T^{-2}$ → matches SI unit newton (N).

Procedure for checking equations

1. Write dimensional formula for each term.
2. Ensure both sides of the equation have identical dimension sets.

3. If mismatch → equation is **dimensionally inconsistent** (common source of errors in JEE problems).

📊 Common JEE Main Question Types

Topic	Typical Ask	Strategy
Unit conversion	Convert $6\ \mathrm{g/cm^3}$ to SI	Memorize conversion factors, keep track of powers of ten
Dimensional check	Verify $E = mc^2$ dimensions	Write $[E] = ML^2T^{-2}$; $[mc^2] = M(LT^{-1})^2 = ML^2T^{-2}$ → correct
Error analysis	Identify computational mistake in a multi-step problem	Re-derive each step, keep intermediate results in symbolic form before plugging numbers
Vector vs scalar	Choose correct representation for a given physical situation	Recognize direction relevance (force, velocity) → vector; magnitude only → scalar

🛠️ Study Strategies Emphasized in the Session

- **Prioritize high-frequency topics**: Unit & Dimension, Error analysis – appear often in JEE Main.
- **Practice with both basic and advanced P&Y Q**: Build confidence from fundamentals to Olympiad-level challenges.
- **Use the Pi App for distraction-free study**: Focused video, built-in notes, instant AI doubt resolution.
- **Regular test series**: Identify weak spots, reduce careless calculation errors.
- **Time management**: Target 4.5–5 hour intensive sessions; allocate remaining time for solving practice questions.

> *Key takeaway*: Mastering the representation, conversion, and dimensional consistency of physical quantities forms the backbone of JEE Main success; leverage the Pi App's integrated resources to practice systematically and eliminate avoidable errors.

📏 Dimensional Formulas – Core Quantities

> *Dimensional formula* expresses a physical quantity as a product of the fundamental dimensions M, L, T raised to integer powers.

1️⃣ Velocity, Acceleration & Derived Forms

Quantity	Symbol	Dimensional Formula	How obtained
Velocity	v	L^1T^{-1}	$v = \frac{\text{displacement}}{\text{time}}$
Acceleration	a	L^1T^{-2}	$a = \frac{v}{t}$ or $a = \frac{\text{change in velocity}}{t}$

2️⃣ Force, Torque & Work

Quantity	Symbol	Dimensional Formula	Relation
Force	F	$M^1L^1T^{-2}$	$F = ma$
Torque	τ	$M^1L^2T^{-2}$	$\tau = F \times (\text{perpendicular dista$
Work / Energy	W, E	$M^1L^2T^{-2}$	$W = F \times \text{displacement}$

> *Note*: Same dimensional formula does **not** guarantee identical physical meaning; torque is a vector, work is a scalar.

3️⃣ Power & Energy Density

Quantity	Symbol	Dimensional Formula
Power	P	$M^1L^2T^{-3}$
Energy Density	u	$M^1L^{-1}T^{-2}$

4️⃣ Common Energy Types (All Share Same Dimensions)

Energy Type	Symbol	Dimensional Formula
Kinetic	K	$M^1L^2T^{-2}$
Potential (gravitational, elastic, electrostatic)	U	$M^1L^2T^{-2}$
Thermal / Heat	Q	$M^1L^2T^{-2}$
Binding (nuclear)	E_b	$M^1L^2T^{-2}$
Rotational kinetic ($\frac{1}{2}I\omega^2$)	K_{rot}	$M^1L^2T^{-2}$

5️⃣ Angle & Solid Angle – Dimensionless Quantities

> *Angle* $\theta = \dfrac{\text{arc length}}{\text{radius}}$ → dimensions $L/L = 1$, **dimensionless**.
> *Unit*: radian (rad).

> *Solid angle* $\Omega = \dfrac{\text{area on sphere}}{r^2}$ → also dimensionless, unit steradian (sr).

🛠️ Frequently Asked Dimensional Forms

Quantity	Symbol	Dimensional Formula
Density (mass)	ρ	M^1L^{-3}
Strain	ε	$L^0 = 1$ (dimensionless)
Pressure / Stress	p	$M^1L^{-1}T^{-2}$ (since $p = F/A$)
Young's Modulus	Y	$M^1L^{-1}T^{-2}$ (same as pressure)
Surface Tension	γ	M^1T^{-2} (force per unit length)
Universal Gravitational Constant	G	$M^{-1}L^3T^{-2}$
Viscosity Coefficient	η	$M^1L^{-1}T^{-1}$

Current Density (a "trick" quantity)	J	Same dimensions as current per area → $M^0L^{-2}T^{-1}$ (note: often confused)

📚 Quick Reference Table – Core Dimensional Formulas

Symbol	Quantity	Dimensional Formula
M	Mass	M^1
L	Length	L^1
T	Time	T^1
v	Velocity	L^1T^{-1}
a	Acceleration	L^1T^{-2}
F	Force	$M^1L^1T^{-2}$
τ	Torque	$M^1L^2T^{-2}$
W, E	Work/Energy	$M^1L^2T^{-2}$
P	Power	$M^1L^2T^{-3}$
θ	Plane angle	1 (dimensionless)
Ω	Solid angle	1 (dimensionless)
p	Pressure/Stress	$M^1L^{-1}T^{-2}$
Y	Young's modulus	$M^1L^{-1}T^{-2}$
γ	Surface tension	M^1T^{-2}
G	Gravitational constant	$M^{-1}L^3T^{-2}$
η	Viscosity	$M^1L^{-1}T^{-1}$
ρ	Mass density	M^1L^{-3}
ε	Strain	1 (dimensionless)

🔄 How to Derive a Dimensional Formula

1. **Write the defining equation** (e.g., $F = ma$).
2. **Replace each symbol with its known dimensions** (mass M, length L, time T).
3. **Combine powers**; ensure the result matches the left-hand side.

> *Example*: For torque, $\tau = F \times r$ → $[F] = MLT^{-2}$, $[r] = L$ → $[\tau] = ML^2T^{-2}$.

📌 Common Pitfalls

- **Same dimensions ≠ same quantity** – torque and work share ML^2T^{-2} but differ in vector/scalar nature.
- **Dimensionless does not imply unitless** – angles have radian units; solid angles have steradian.
- **Force-related quantities** – many forces (gravitational, normal, tension, centrifugal, etc.) all have MLT^{-2}; remember to treat the context separately.
- **Energy-density vs. energy** – dividing energy by volume adds L^{-3}, changing ML^2T^{-2} to $ML^{-1}T^{-2}$.

🧮 Sample Dimensional Checks

1. **Gravitational constant G**
   - From $F = G\frac{m_1m_2}{r^2}$ → $[G] = \frac{[F], [r]^2}{[m]^2} = \frac{MLT^{-2}, L^2}{M^2} = M^{-1}L^3T^{-2}$.
2. **Surface tension γ**
   - Defined as force per unit length: $\gamma = \frac{F}{L}$ → $[\gamma] = \frac{MLT^{-2}}{L} = MT^{-2}$.
3. **Power P from work**
   - $P = \frac{W}{t}$ → $[P] = \frac{ML^2T^{-2}}{T} = ML^2T^{-3}$.

Keep this sheet handy for quick dimensional verification during JEE Main problem solving.

📏 Density Variants & Dimensional Distinctions

> **Mass density** – mass divided by volume.
> **Energy density** – energy divided by volume.

- Mass density dimension: $[\rho] = M, L^{-3}$ (as derived earlier).

- Energy density dimension: $[u] = M, L^{-1}, T^{-2}$ because energy $[E] = M, L^2, T^{-2}$ and volume L^3 gives M, L^{-1}, T^{-2} .

🧪 12th-Grade Physical Quantities (Quick Reference)

Quantity	Symbol	Typical Formula	Dimension
Permittivity	ε	–	M^{-1}, L^{-3}, T^4, I^2
Electric current	I	–	I
Planck constant	h	–	M, L^2, T^{-1}
Frequency	ν	–	T^{-1}
Gravitational acceleration	g	–	L, T^{-2}
Coefficient of friction	μ	–	dimensionless
Coefficient of viscosity	η	–	M, L^{-1}, T^{-1}

🔧 Planck Relation & Dimensional Check

The relation taught was

$$E = h, \nu$$

- $[E] = M, L^2, T^{-2}$
- $[h] = M, L^2, T^{-1}$
- $[\nu] = T^{-1}$

Multiplying $[h][\nu]$ reproduces $[E]$, confirming dimensional consistency.

🌀 Viscosity & Stokes' Law

Stokes' law for a sphere moving in a viscous fluid:

$$F_{\text{visc}} = 6\pi, \eta, r, v$$

- 6π is dimensionless.

- $[\eta] = M, L^{-1}, T^{-1}$ (coefficient of viscosity).
- $[r] = L$ (radius).
- $[v] = L, T^{-1}$ (velocity).

Hence

$$[F_{\text{visc}}] = M, L, T^{-2}$$

which matches the dimension of force.

🔄 Torque, Radius of Gyration & Moment of Inertia

- **Moment of inertia (point mass)**: $I = m, r^2$ → $[I] = M, L^2$.
- **Radius of gyration** k is defined via $I = m, k^2$ → $[k] = L$.
- **Torque**: $\tau = F, r$ → $[\tau] = M, L^2, T^{-2}$.

> Note: Torque and work share the same dimension M, L^2, T^{-2} but differ in physical nature (vector vs scalar).

🌡️ Gradients (Temperature, Potential, etc.)

A gradient is expressed as a change of a quantity per unit length:

$$\text{gradient} = \frac{\Delta Q}{\Delta x}$$

- Dimension of temperature gradient: $[, \Delta T / \Delta x,] = \Theta, L^{-1}$ (since temperature Θ is a fundamental dimension).
- The same rule applies to potential gradient, concentration gradient, etc.: dimension = (dimension of quantity) × L^{-1} .

🌍 Gravitational Field & Acceleration

Gravitational field $\mathbf{g}$ is force per unit mass:

$$\mathbf{g} = \frac{\mathbf{F}}{m}$$

- $[g] = L, T^{-2}$ (same as acceleration).

> The lecture emphasised that any “field” defined as force divided by a scalar (mass or charge) inherits dimensions of acceleration or electric field accordingly.

⚙️ Coefficient of Friction

Frictional force: $F_{\text{fr}} = \mu, N$ (normal force N).

- μ is **dimensionless**; it carries no units.

🔥 Thermal Conductivity

Fourier's law (steady-state conduction):

$$\frac{dq}{dt} = k, A, \frac{\Delta T}{L}$$

- $[dq/dt] = M, L^2, T^{-3}$ (power).
- $[A] = L^2$, $[\Delta T] = \Theta$, $[L] = L$.

Solving for k:

$$[k] = \frac{[dq/dt]}{[A],\Delta T/L} = \frac{M,L^2,T^{-3}}{L^2,\Theta,L^{-1}} = M, L^{-1}, T^{-3}, \Theta^{-1}$$

📐 Surface Charge Density

Surface charge density σ is charge per unit area:

$$\sigma = \frac{Q}{A}$$

- $[Q] = I, T$, $[A] = L^2$ → $[\sigma] = I, T, L^{-2}$.

If the "mass surface density" is needed, replace Q with mass M:

$$\sigma_m = \frac{M}{A};; \Rightarrow;; [\sigma_m] = M, L^{-2}$$

📊 Dimensional Rules Recap (From Earlier Sections)

1. **Addition / Subtraction** – only quantities with identical dimensions may be combined.
2. **Multiplication / Division** – dimensions combine algebraically (add/subtract exponents).
3. **Constants** – pure numbers (e.g., 6π) are dimensionless and do not affect the overall dimension.

4. **Derived quantities** – obtain dimensions by substituting fundamental dimensions into defining equations (as demonstrated for viscosity, torque, etc.).

📈 Impulse & Angular Impulse

- **Linear impulse**: $J = F, \Delta t$ → $[J] = M, L, T^{-1}$.
- **Angular impulse**: $\mathcal{J} = \tau, \Delta t$ → $[\mathcal{J}] = M, L^2, T^{-1}$.

Both are useful in JEE-Main problems involving collisions and rotational dynamics.

🧊 Latent Heat

Latent heat L relates heat transfer to mass change:

$$\Delta Q = m, L$$

- $[\Delta Q] = M, L^2, T^{-2}$ (energy).
- $[m] = M$.

Thus

$$[L] = \frac{[\Delta Q]}{[m]} = L^2, T^{-2}$$

The dimension is identical to that of specific energy (energy per unit mass).

🔗 Dimensional Consistency in Composite Expressions

> *Dimensional consistency* – every term that is added or subtracted in an equation must have **exactly the same dimensional formula**.

- When two quantities **join** (addition/subtraction) their dimensions are *identical*.
- When two quantities **multiply** or **divide** their dimensions combine **algebraically** (add/subtract exponents of M, L, T).

Quick rule-sheet

Operation	Effect on dimensions
$A + B$ or $A - B$	$[A] = [B]$ (must match)
$A \times B$	$[A], [B]$ (multiply exponents)

$\frac{A}{B}$	$[A], [B]^{-1}$ (subtract exponents)

Example from the lecture

Force F was written as

$$F = a, x^2 + b, t^{1/2}$$

- $[F] = MLT^{-2}$ (force).
- $[x] = L$, so $[a, x^2] = [a], L^2$.
- $[t] = T$, so $[b, t^{1/2}] = [b], T^{1/2}$.

Setting each term equal to $[F]$:

$$[a], L^2 = MLT^{-2};; \Rightarrow;; [a] = ML^{-1}T^{-2} \quad [b], T^{1/2} \quad = MLT^{-2};; \Rightarrow;; [b] = MLT^{-5/2}$$

Thus

- a carries dimensions $ML^{-1}T^{-2}$ (same as **pressure/ stress**).
- b carries dimensions $MLT^{-5/2}$.

Solving for Unknown Constants

The lecture repeatedly asked students to **solve for the dimensional formula of unknown coefficients** (denoted a, b, α, β, etc.) by matching terms. Follow these steps:

1. **Identify the target quantity's dimension** (e.g., force $= MLT^{-2}$, energy $= ML^2T^{-2}$, power $= ML^2T^{-3}$).
2. **Write the dimensional expression of each term** using known symbols (x, t, v, etc.).
3. **Equate the exponent of each fundamental dimension** on both sides of the equality.
4. **Solve the resulting algebraic equations** for the unknown exponent(s) of the coefficient.
5. **Write the final dimension of the coefficient**.

Worked illustration

Given

$$u = \frac{ax^2}{b+t^2}$$

- u is **energy** → $[u] = ML^2T^{-2}$.

- $[x] = L, [t] = T$.

Assume $[a] = M^p L^q T^r$ and $[b] = M^s L^y T^z$.

$\frac{[a],L^2}{[b]+T^2} = ML^2T^{-2}$

Since $[b]$ and T^2 must have the **same dimensions** (they are added),

$[s], L^y, T^z = T^2;; \Rightarrow;; s = 0,; y = 0,; z = 2$

Hence $[b] = T^2$ (dimensionless mass and length).

Now match the numerator:

$[a], L^2 = ML^2T^{-2};; \Rightarrow;; [a] = MT^{-2}$

So

- a → MT^{-2} (same as **force per unit length**).
- b → T^2 .

⚙️ Common JEE-Main Patterns & Quick Checklists

Question type	Typical expression	Dimensional check steps
Force expressed with mixed powers	$F = a, x^n + b, t^m$	1. Set $[F] = MLT^{-2}$. 2. Write $[a]L^n$ and $[b]T^m$. 3. Equate each to MLT^{-2} to find $[a]$ and $[b]$.
Energy / Work with denominator	$E = \frac{k, x^2}{t}$	1. $[E] = ML^2T^{-2}$. 2. $[k]L^2T^{-1} = ML^2T^{-2}$ → $[k] = MT^{-1}$.
Pressure / Stress from force/area	$p = \frac{F}{A}$	$[p] = ML^{-1}T^{-2}$. Use $[F]$ and $[A] = L^2$ to verify.
Power from work/time	$P = \frac{W}{t}$	$[P] = ML^2T^{-3}$. Check $[W] = ML^2T^{-2}$.
Angular impulse	$\mathcal{J} = \tau, \Delta t$	$[\tau] = ML^2T^{-2}$, so $[\mathcal{J}] = ML^2T^{-1}$.
Viscosity coefficient	η appears in $F_{\text{visc}} = 6\pi\eta r v$	Verify $[\eta] = ML^{-1}T^{-1}$ by cancelling $[r] = L, [v] =$

		LT^{-1} .

"Spot-the-Error" checklist (useful during a timed exam)

- **Add/subtract**: Do *all* terms share the same M, L, T powers?
- **Cancel correctly**: When moving a factor across the equality sign, flip its exponent sign.
- **Watch for hidden squares/cubes**: x^2 contributes L^2, $t^{1/2}$ contributes $T^{1/2}$.
- **Dimensionless constants** (e.g., 6π) do not affect dimensions.

📚 Reference Table – Frequently Solved Coefficients

Symbol	Appears in	Solved dimension (from lecture)
a	$F = a, x^2 + \dots$	$ML^{-1}T^{-2}$
b	$F = \dots + b, t^{1/2}$	$MLT^{-5/2}$
α	$W = \alpha$ (work)	ML^2T^{-2}
β	denominator $b + t^2$	T^2
k	$E = k, x^2/t$	MT^{-1}
η	Stokes' law $F_{\text{visc}} = 6\pi\eta rv$	$ML^{-1}T^{-1}$

Use this table as a **quick lookup** while practicing JEE-Main dimensional-analysis questions.

> **Key practice tip:** Write a one-line dimensional equation for every unknown coefficient before plugging numbers. This prevents careless exponent errors that commonly appear in JEE-Main's time-pressured sections.

🔁 Repetitive Question Patterns

> *If a question type appears repeatedly in past papers, its probability of re-appearing is high.*

- Keep an eye on **angular momentum, latent heat, torque, capacitor, inductor, resistivity, Bohr-Sommerfeld constant, coefficient of linear expansion, Planck's constant, thermal conductivity** – all asked in JEE Advanced (e.g., 2013).

- Mark these as **high-yield** topics; allocate extra revision time.

📚 Core Topics Emphasized for JEE Main & Advanced

Category	Example Items	Reason for focus
Fundamental constants	Bohr-Sommerfeld constant, Planck constant, Stefan-Boltzmann constant	Frequently appear in both unit-matching and conceptual questions.
Energy-related quantities	Latent heat, torque, work, power	Appear in dimensional-analysis and unit-conversion problems.
Electrical quantities	Capacitance, inductance, resistivity, coefficient of viscosity	Common in JEE Advanced "match the column" format.
Thermodynamics	Thermal conductivity, coefficient of linear expansion	Appear in JEE Main as direct numerical questions.

> *Prioritise these during practice tests; they dominate the high-frequency question pool.*

📐 Dimensional Behaviour of Trigonometric, Exponential & Logarithmic Functions

> *Any argument of $\sin$, $\cos$, $\tan$, $\log$, $\ln$ or the exponent of e must be* ***dimensionless****.*

- **Why?**
  - Trigonometric ratios are **ratios of lengths** → dimension cancels → **dimensionless**.
  - $\log$ and $\ln$ are defined for pure numbers → **dimensionless**.
  - Exponential e^{anything} requires a pure number in the exponent.
- **Practical rule:**
  1. Treat the whole argument (the "kaddoo") as **$=1$** in dimensional analysis.

2. Consequently, the coefficient multiplying the argument carries the opposite dimensions so that the product becomes dimensionless.

Example

Given (y = a\sin(bx^{2}) + c)

- (\sin) argument (bx^{2}) → dimensionless ⇒ ([b],[x]^{2}=1)
    - If (x) is a **length** ([x]=L) → ([b]=L^{-2})
- Constant term (c) must also be dimensionless ⇒ ([c]=1) (or a pure number).

Thus

$$[a] = [y] \quad (\text{dimension of } y), \qquad [b] = L^{-2}, \qquad [c] = 1$$

🧮 Solving for Coefficients in Composite Expressions

General Procedure

1. **Write the full expression.**
2. **Identify the physical quantity on the left-hand side (LHS)** and write its dimension ([LHS]).
3. **For each term on the RHS**, replace variables with their known dimensions.
4. **Impose dimensional-lessness** on any function arguments (trig, log, exponential).
5. **Equate exponents of (M, L, T)** on both sides to obtain dimensions of unknown coefficients.

Worked Example

Expression:

$$y = a\sin(bx^2) + c,t$$

- LHS: assume (y) is a **length** → ([y]=L).
- (x) is a length ([x]=L); (t) is time ([t]=T).

Step 1 – Trig argument:
([b],L^{2}=1 ;\Rightarrow; [b]=L^{-2}).

Step 2 – Second term:
([c],T = L ;\Rightarrow; [c]=L,T^{-1}).

Step 3 – Coefficient (a):
Since (\sin(\dots)) is dimensionless, the first term contributes ([a]) directly:
([a]=L).

Result:

Symbol	Dimension
(a)	(L)
(b)	(L^{-2})
(c)	(L,T^{-1})

Shortcut for Repeated Patterns

- Replace **any** (\sin, \cos, \tan, \log, \ln, e^{(\cdot)}) argument with **1**.
- The coefficient in front of the argument then gets the **inverse** dimension of the argument's base quantity.

📊 Expected Question Types & Rapid Solving Strategies

Question Type	Typical Form	Fast-track Method
Match-the-Column (Units ↔ Formula)	Formula in LHS, unit in RHS	Verify dimensions of each formula; match with given unit.
Coefficient-Finding (e.g., (a, b) in composite expressions)	(y = a\sin(bx^{2}) + c)	Apply the *dimensionless-argument* rule; solve linear equations for exponents.
Dimensionless-Function Substitution	Replace (\sin) by (\cos) or (\log) by (\ln)	Answer remains unchanged; focus only on coefficient dimensions.
Energy-Related Algebra (e.g., (W = \alpha/\beta;e^{-\gamma}))	Work, power, pressure appear with exponentials	Treat exponentials as dimensionless; set ([\alpha]=[W],[\beta]) and solve.
Boltzmann / Bohr-Sommerfeld Constants	(U = \frac{3}{2}k_{B}T)	Recognise (k_{B}T) as energy → dimension (M L^{2} T^{-2}).

Quick Check-List for Each Problem

1. **Identify the target quantity** and write its dimension.
2. **Mark every function argument** (trig, log, exponent) as dimensionless.
3. **Write dimensional equations** for unknown coefficients.

4. **Solve exponent equations** (add/subtract powers of (M, L, T)).
5. **Validate** by ensuring LHS and RHS dimensions match exactly.

📚 Consolidated Formula Sheet (Attached in PPT Format)

- The instructor has provided **1-21** core formulas (including gas constant, Stefan-Boltzmann constant, etc.).
- **Action:** Review each formula, note its dimension, and practice matching with its unit.
- **Tip:** For constants that appear with temperature (e.g., (k_{B}T)), treat the product as **energy** ((M L^{2} T^{-2})).

> *Mastering the dimensionless-argument rule and the systematic coefficient-solving workflow dramatically reduces calculation time, especially in the 5-minute "match-the-column" JEE Advanced questions.*

🔎 Advanced Coefficient Determination in Force Expressions

> *When a physical expression contains unknown coefficients, write the dimensional formula of every factor, impose that the whole expression has the dimension of the quantity on the left-hand side, and solve for the unknowns.*

Example discussed

Expression given (simplified):

$$F = a, \frac{d}{b}, \cos(\theta)$$

- (F) – force (\displaystyle [F]=M L T^{-2})
- (\cos(\theta)) – **dimensionless** (argument of any trigonometric, logarithmic or exponential function must be dimensionless).

Steps followed in the lecture:

1. **Identify dimensions of each symbol**
   - (d) appeared as a time-derivative factor ⇒ ([d]=T^{-1}) (power of (t) is (-1)).
   - (b) appeared with length ⇒ ([b]=L^{-1}).
2. **Form the dimensional equation**

$[MLT^{-2}] = [a]; \frac{[d]}{[b]}; (1)$

3. **Substitute known dimensions**

$[MLT^{-2}] = [a]; \frac{T^{-1}}{L^{-1}} = [a]; LT^{-1}$

4. **Solve for ([a])**

$[a] = \frac{MLT^{-2}}{LT^{-1}} = MT^{-1}$

Hence

$$\boxed{[a] = M, T^{-1}}, \qquad [b] = L^{-1}, \qquad [d] = T^{-1}$$

The result matched the **second option** of the multiple-choice question presented in class.

📐 Integration Symbols and Their Dimensional Meaning

> *Differential symbols are not "new" quantities; they inherit the dimension of the variable they differentiate.*

Symbol	Physical meaning	Dimension
(dx)	infinitesimal **length** change	(L)
(dt)	infinitesimal **time** change	(T)
(df)	infinitesimal **force** change	(M L T^{-2})
(dw)	infinitesimal **work/energy** change	(M L^{2} T^{-2})
(\Delta x)	finite length change (same as (dx))	(L)
(\Delta t)	finite time change (same as (dt))	(T)

> *Never let the presence of a differential symbol make you think a new dimension appears; simply replace it by the dimension of the underlying variable.*

📊 Profiles of Typical JEE Main/Advanced Dimensional Questions

Profile	Typical Prompt	Core Strategy
1️⃣ **Direct-Formula Dimensional Check**	"Find the dimensional formula of the given expression."	Write dimensions of each factor, multiply/divide, compare with required quantity.
2️⃣ **Match-the-Column / Fill-in-Blank**	"Match symbols to their correct dimensions or units."	Prepare a quick reference table (see earlier sections) and use elimination.
3️⃣ **Variable-Expression (e.g., (x = at + bt^{2}))**	"Determine the dimensions of constants (a, b)."	Treat each term as having the same dimension as the left-hand side; solve linear equations for exponents.
**4️⃣ Formula-Derivation (Dependency) **	"Assume a quantity depends on (r, M, G); find its dimensional formula."	Assume (Q \propto r^{x} M^{y} G^{z}); write dimensions, equate powers of (M, L, T); solve for (x,y,z).
5️⃣ **Fundamental-Quantity Substitution**	"Express mass in terms of force, length, velocity, or energy."	Use known dimensional formulas (e.g., (F = M L T^{-2}), (v = L T^{-1}), (E = M L^{2} T^{-2})) to replace the unknown.
6️⃣ **Unit-Conversion**	"Convert 1 N to dynes, 1 J to ergs, etc."	Apply conversion factors for each base unit (kg ↔ g, m ↔ cm, s unchanged); adjust powers of ten accordingly.

🧪 Solving a Composite Expression (Worked Example)

Given in class:

$$\frac{a,d}{b} = F \quad \text{with} \quad d = t^{-1},; b = L^{-1}$$

1. Write dimensions: ([a], [d] / [b] = M L T^{-2}).
2. Substitute ([d]=T^{-1}), ([b]=L^{-1}):

$[a]; \frac{T^{-1}}{L^{-1}} = [a], L, T^{-1}$

3. Equate to force dimension:

$[a], L, T^{-1} = MLT^{-2};; \Rightarrow;; [a] = MT^{-1}$

Thus **(a) carries the dimension of momentum per unit time** (i.e., force).

Dimensional Derivation of Orbital Velocity

Problem: *Find the dimensional expression for the orbital velocity (v) of a satellite around a planet, assuming it depends only on the orbital radius (r), the planet's mass (M), and the universal gravitational constant (G).*

Steps (as illustrated)

1. **Assume a power-law form**

$v; \propto; r^x, M^y, G^z$

2. **Write dimensions**

$[v] = LT^{-1} \; [r^x] = L^x \; [M^y] = M^y \; [G^z] = \left(M^{-1}L^3T^{-2}\right)^z = M^{-z}L^{3z}T^{-2z}$

3. **Combine**

$LT^{-1} = M^{,y-z}; L^{,x+3z}; T^{-2z}$

4. **Equate exponents for each fundamental dimension**

$\left\{ M :; y - z = 0 \; L :; x + 3z = 1 \; T :; -2z = -1 \right.$

5. **Solve**

$z = \frac{1}{2}, \quad y = z = \frac{1}{2}, \quad x = 1 - 3z = 1 - \frac{3}{2} = -\frac{1}{2}$

6. **Resulting dimensional formula**

$v; \propto; \frac{\sqrt{GM}}{\sqrt{r}}$

> *The constant of proportionality is dimensionless; the final expression matches the well-known orbital speed (v = \sqrt{GM/r}).*

💧 Fluid-Resistance Example (Stokes' Law)

Expression:

$$F_{\text{visc}} = 6\pi, \eta, r, v$$

- (F_{\text{visc}}) – force ([F]=M L T^{-2})
- (\eta) – coefficient of viscosity ([\eta]=M L^{-1} T^{-1})
- (r) – radius ([r]=L)
- (v) – velocity ([v]=L T^{-1})

Dimensional check:

$$[\eta][r][v] = (ML^{-1}T^{-1})(L)(LT^{-1}) = MLT^{-2}$$

The product reproduces the force dimension, confirming the law's dimensional correctness.

🔄 Quick Unit-Conversion Cheat Sheet

Quantity	SI Unit	CGS Equivalent	Conversion Factor
Force	newton (N) = kg·m·s(^{-2})	dyne (dyn) = g·cm·s(^{-2})	(1;\text{N}=10^{5};\te
Energy	joule (J) = kg·m(^2)·s(^{-2})	erg = g·cm(^2)·s(^{-2})	(1;\text{J}=10^{7};\te
Power	watt (W) = kg·m(^2)·s(^{-3})	ergs · s(^{-1})	(1;\text{W}=10^{7};\t

> *When converting, replace each SI base unit with its CGS counterpart (kg→g, m→cm) and adjust the overall factor (10^{n}) accordingly.*

📏 Handling Dimensionless Arguments

> *Any argument of (\sin, \cos, \tan, \log, \ln, e^{(\cdot)}) must be pure number (dimensionless).*

- **Procedure**:
    1. Write the argument's dimensional expression.
    2. Set its overall dimension equal to (1).

3. Solve for the unknown coefficient(s) that appear multiplicatively.

Illustration (from lecture):

$$y = a\sin(bx^2) + c$$

- ([y]) – given (e.g., length (L)).
- Argument: (bx^{2}) → ([b],L^{2}=1 \Rightarrow [b]=L^{-2}).
- First term contributes ([a]) directly (since (\sin) is dimensionless).
- Second term (c) must have the same dimension as (y) → ([c]=L).

Resulting dimensions:

$$[a] = L, \quad [b] = L^{-2}, \quad [c] = L$$

📚 Symbol-Reference Table for Differential Notations

Symbol	Meaning in Physics	Dimension
(dx)	infinitesimal displacement	(L)
(dt)	infinitesimal time interval	(T)
(df)	infinitesimal force change	(M L T^{-2})
(dw)	infinitesimal work/energy change	(M L^{2} T^{-2})
(\Delta x)	finite change in position	(L)
(\Delta t)	finite change in time	(T)

> *Treat all (\Delta) and (d) symbols as carrying the same dimension as the variable they act upon.*

🧩 Summary of Problem-Solving Workflow

1. **Identify the target quantity** and write its dimension.
2. **List all symbols** appearing in the expression, assigning known dimensions where possible.
3. **Force any function arguments** (trig, log, exp) to be dimensionless → solve for coefficients inside them.

4. **Set up a dimensional equality** between LHS and RHS; equate exponents of (M, L, T).
5. **Solve the resulting algebraic system** for unknown exponents or coefficient dimensions.
6. **Verify** that the final expression is dimensionally consistent; if not, revisit steps 2-4.

Following this systematic checklist eliminates the common "dimension-mismatch" errors that frequently appear in JEE Main and Advanced exams.

🧩 Common Pitfalls & Mindset

> *Only ~ 3–4 % of students finish a JEE Main paper with all answers correct.*
> The difference lies in **knowing the method** versus **actually executing it** on paper.

- First-attempt errors usually stem from **incorrect exponent handling** (e.g. writing density as $m\ell^x$ instead of m/ℓ^3).
- Success rate for these "power-type" questions is **≤ 10 %**, so systematic practice is mandatory.

📐 Dimensional Analysis – Solving for Unknown Exponents

Method 1: Full-Equation Approach

1. Write the **dimensional formula** of the target quantity.
2. Express every factor on the RHS with its own powers of M, L, T .
3. Equate the exponents of M, L, T on both sides → solve for the unknown powers (often denoted x, y, z).

Step	Example (density)
Target	$\rho = \frac{m}{\ell^3} \rightarrow [\rho] = ML^{-3}$
RHS assumed	$m^{,a}, \ell^{,b}, s^{,c} \rightarrow [m]^a[,\ell]^b[,s]^c = M^aL^bT^c$
Equate	$a = 1, b = -3, c = 0$
Result	$a = 1, b = -3, c = 0$ (matches the known formula)

Method 2: Missing-Dimension Zero Rule

- If a **fundamental quantity is absent** on either side, its exponent **must be zero** on that side.
- This quickly eliminates terms without solving a full system.

> *"If mass is missing in the LHS, it must also be missing in the RHS; set its exponent to 0."*

Quick-Check Table

Quantity	Present in LHS?	Action
Mass M	No	Set exponent of M on RHS to 0
Length L	Yes	Keep exponent variable (solve normally)
Time T	No	Set exponent of T on RHS to 0

🌊 Wave-Speed in Water – Worked Example

Given:

- v (wave speed) = ?
- λ (wavelength) = length → $[\lambda] = L$
- a – dimensionless coefficient (power of λ)
- g – acceleration → $[g] = LT^{-2}$
- ρ – mass density → $[\rho] = ML^{-3}$

Assume:
$v = \lambda^{,a}, g^{,b}, \rho^{,c}$

Symbol	Dimensional Formula
v	LT^{-1}
λ^a	L^a
g^b	$(LT^{-2})^b = L^bT^{-2b}$
ρ^c	$(ML^{-3})^c = M^cL^{-3c}$

Equate exponents:

$$\begin{cases} L :; 1 = a + b - 3c \; T :; -1 = -2b \; M :; 0 = c \end{cases} \implies b = \frac{1}{2},; c = 0,; a = 1 - b = \frac{1}{2}$$

Result:

$$v = \lambda^{1/2}, g^{1/2} \quad (\text{since } \rho^0 = 1)$$

The answer matches the **options** where $a = b = \frac{1}{2}, c = 0$.

🔧 Unit-Conversion in a Custom System

System definition:

Unit	Value in SI
Mass M_u	2 kg
Length L_u	5 m
Time T_u	5 s

Task: Find 1 unit of force F_u in newtons.

1. Dimensional formula of force: $[F] = MLT^{-2}$.
2. Substitute the unit values:

$$F_\text{u} = \frac{M_\text{u},L_\text{u}}{T_\text{u}^2} = \frac{2 \text{ kg} \times 5 \text{ m}}{5^2 \text{ s}^2} = \frac{10}{25} \text{ kg}\cdotpm\cdotps^{-2} = \frac{2}{5} \text{ N}$$

Hence

$$1 \text{ N} = \frac{5}{2} \, F_\text{u}$$

📚 Practice Recommendations

- **Minimum practice:** solve ≥ 4 questions of the "power-type" (unknown exponents) before the exam.
- **Success-ratio reminder:** only ~ 10 % of candidates nail these on the first attempt.
- **Strategy checklist:**
  1. Write the target dimensional formula.
  2. Identify which fundamental quantities appear (M, L, T).
  3. Apply the **Missing-Dimension Zero Rule** to discard unnecessary variables.

4. Set up exponent equations, solve, and verify by substituting back.
5. If an option matches the derived exponents, confirm by a quick plug-in.

- **When to pause:** before starting a calculation, ensure the problem isn't a straightforward substitution (e.g., density = M/L^3) that can be answered instantly.

⚙️ General Solution Framework for Expressions like

$$\text{Quantity} = (\text{Force})^x, (\text{Velocity})^y, (\text{Time})^z$$

1. **List dimensions**
   - $[F] = MLT^{-2}$
   - $[v] = LT^{-1}$
   - $[t] = T$
2. **Combine:**

   $$[\text{RHS}] = M^x L^x, T^{-2x}; \times; L^y T^{-y}; \times; T^z = M^x L^{x+y} T^{-2x-y+z}$$

3. **Equate with LHS dimension** (e.g., density ML^{-3}):

   $$\left\{ M: \quad x = 1 \; L: \; x + y = -3 \; T: \quad -2x - y + z = 0 \right.$$

4. **Solve** → obtain x, y, z .
5. **Check** that any missing fundamental (e.g., M) has exponent 0 on both sides.

Following this framework repeatedly builds speed and accuracy for the high-frequency JEE Main dimensional-analysis questions.

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> □□□□□□□□□□ □□□□□□□□□□ : $[E] = M, L^2, T^{-2}$, $[F] = M, L, T^{-2}$

- **□□□□□ □□ □□□□□□□**
  - □□□□ □□□: $m = 2$ kg, $l = 5$ m, $t = 5$ s
  - $E = m, l^2, t^{-2} = 2 \times 5^2 \times 5^{-2} = 2$ J
  - □□□ □□ □□□□□□□ □□□ $1\ \text{J}{\text{new}}=2\ \text{J}{\text{SI}}$

- [illegible] (Force) [illegible]
  - [illegible] $n_1 u_1 = n_2 u_2$ [illegible]
  - $n_1 = 100\ (\mathrm{N}), u_1 = \mathrm{N}$ (SI)
  - u_2 = [illegible] 1 N [illegible]
  - $n_2 = \frac{n_1 u_1}{u_2} = \frac{100 \times 1}{\frac{5}{2}} = 250\ \mathrm{N_{new}}$
- [illegible] : 1 N ([illegible]) = $\frac{5}{2}$ N (SI)

2️⃣ [illegible] → [illegible]

> *[illegible]* = "[illegible]", $1\ \mathrm{cal} = 4.2\ \mathrm{J}$

- [illegible] $1\ \mathrm{cal} = 4.2\ \mathrm{J}$ [illegible] m, l, t [illegible]:
  - $[\mathrm{cal}] = M, L^2, T^{-2}$ ([illegible])
  - $m = \alpha\ \mathrm{kg}, l = \beta\ \mathrm{m}, t = \gamma\ \mathrm{s}$ → $4.2 = \alpha^{-1}, \beta^{-2}, \gamma^2$

📏 12[illegible] – [illegible]

3️⃣ [illegible] (Q) [illegible] (I)

> *[illegible]*: [illegible] = $Q = A, t$ ([illegible] × [illegible])
> *[illegible]*: [illegible] = I ([illegible] = [illegible])

[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	Q	I, T	C ([illegible])
[illegible]	I	I	A ([illegible])

4️⃣ [illegible] ($\mathbf{E}$), [illegible] (V), [illegible] (R)

- $\mathbf{E} = \frac{F}{Q}$ → [illegible] $[E] = M, L, T^{-3}, I^{-1}$
- [illegible] $V = \frac{U}{Q}$ → [illegible] $[V] = M, L^2, T^{-3}, I^{-1}$

- □□□□□□□□□□ $R = \dfrac{V}{I}$ → □□□□ $[R] = M, L^2, T^{-3}, I^{-2}$

□□□□□□□□□□	□□□□□	□□□□
□□□□□□□□□□ □□□□□□	$E = \frac{F}{Q}$	M, L, T^{-3}, I^{-1}
□□□□□□□□□	$V = \frac{U}{Q}$	M, L^2, T^{-3}, I^{-1}
□□□□□□□□□□	$R = \frac{V}{I}$	M, L^2, T^{-3}, I^{-2}

5️⃣ □□□□ □□□□□□□□ (J), □□□□□□□□□□ □□□□□□ (Φ_E), EMF ($\mathcal{E}$)

- $\mathbf{J} = \sigma\mathbf{E}$ → □□□□ $[J] = I, L^{-2}$
- □□□□□□□□□□ □□□□□□ $\Phi_E = \mathbf{E} \cdot \mathbf{A}$ → □□□□ $[\Phi_E] = M, L, T^{-3}, I^{-1}$
- EMF = □□□□□□□□□ □□□□ = $\mathcal{E} = V$ → □□□□ □□□ $[V]$

6️⃣ □□□□□□□□□□ (C) □□ □□□□□□□□□□□ (L)

- $C = \dfrac{Q}{V}$ → □□□□ $[C] = I^2, T^4, M^{-1}, L^{-2}$
- □□□□□□□□□□ $L = \dfrac{\Phi_B}{I}$ → □□□□ $[L] = M, L^2, T^{-2}, I^{-2}$

7️⃣ □□□□□□□□□□□□ (ρ) □□ □□□□□□□□□□□□ (σ)

- $\rho = \dfrac{R, A}{l}$ → □□□□ $[\rho] = M, L^3, T^{-3}, I^{-2}$
- $\sigma = \dfrac{1}{\rho}$ → □□□□ $[\sigma] = M^{-1}, L^{-3}, T^3, I^2$

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8️⃣ □□□□□□□□□□ □□□□□□ (B) □□ □□□□□□□ □□

- □□□□□□□ □□□□□□□□□□ $\mathbf{F} = q, \mathbf{v} \times \mathbf{B}$ → □□□□ $[B] = M, T^{-2}, I^{-1}$
- □□□ $F = qvB$ → $[B] = \dfrac{M, L, T^{-2}}{I, L, T^{-1}} = M, T^{-2}, I^{-1}$

9️⃣ □□□□□□□-□□□□□□□ □□□ □□□□□ $E = c, B$

- c = □□□□□□ □□□, □□□□ $[c] = L, T^{-1}$
- □□ □□□□□ □□ $[E] = [c][B]$ □□ □□□□ □□□□ □□ (□□□□□ □□□□ □□□□)

🔟 ⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠ (u)

- ⊠⊠⊠⊠⊠⊠⊠: $u = \frac{▯▯▯▯▯}{▯▯▯▯}$ → ⊠⊠⊠⊠ $[u] = M, L^{-1}, T^{-2}$
- ⊠⊠⊠⊠⊠⊠: $u = \frac{1}{2}, \varepsilon_0 E^2$ ⊠⊠ $u = \frac{B^2}{2\mu_0}$ (⊠⊠⊠⊠⊠ ⊠⊠ ⊠⊠⊠⊠ M, L^{-1}, T^{-2})

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⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠	$F = qvB$	M, L, T^{-2}
⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠	$u = \frac{E}{V}$	M, L^{-1}, T^{-2}
EM-⊠⊠⊠ ⊠⊠⊠⊠⊠	$E = cB$	$[E] = [c][B]$

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11️⃣ $n_1 u_1 = n_2 u_2$ ⊠⊠ ⊠⊠⊠⊠⊠⊠

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⊠⊠⊠⊠⊠⊠ – 100 N → $n_2 = 250\ \mathrm{N_{new}}$ (⊠⊠⊠⊠ ⊠⊠⊠ ⊠⊠⊠⊠⊠⊠ ⊠⊠⊠)⊠

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- ⊠⊠⊠⊠⊠⊠⊠⊠ ⊠⊠⊠ M, L, T ⊠⊠ ⊠⊠⊠ ⊠⊠ ⊠⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠⊠⊠ – ⊠⊠⊠⊠ ⊠⊠⊠⊠⊠⊠ ⊠⊠⊠ ⊠⊠⊠⊠ ⊠⊠⊠ ⊠⊠⊠⊠⊠ ⊠⊠⊠⊠ ⊠⊠⊠⊠⊠
- ⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠⊠, ⊠⊠⊠⊠, ⊠⊠⊠⊠⊠⊠ ⊠⊠ ⊠⊠⊠⊠⊠ ⊠⊠⊠⊠ ⊠⊠⊠ ⊠⊠⊠⊠:
  - $[F] = M, L, T^{-2}$, $[U] = M, L^2, T^{-2}$, $[J] = I, L^{-2}$ ⊠⊠⊠⊠

- [illegible] “[illegible] = k SI-units” [illegible]; [illegible]

[illegible] ([illegible])	k (SI [illegible])	[illegible]
1 N ([illegible])	$\frac{5}{2}$ N (SI)	[illegible]-[illegible]
1 J ([illegible])	2 J (SI)	[illegible]-[illegible]
1 cal ([illegible])	4.2 J (SI)	[illegible]-[illegible]

Magnetic Flux & Inductance

Definition of Magnetic Flux

> *Magnetic flux* $(\Phi) = B, A$ where B is the magnetic field and A is the area normal to the field lines.

Self-Inductance

$L; =; \frac{\Phi}{I}$

- Φ = magnetic flux linked with the circuit.
- I = current through the coil.

Dimensional formula

$[\Phi] = [B]L^2 = M, T^{-2}, I^{-1} L^2, \qquad [L] = \frac{[\Phi]}{[I]} = M, L^2, T^{-2}, I^{-2}$

Mutual Inductance

$M; =; \frac{\Phi_{12}}{I_2}$

- Φ_{12} = flux in circuit 1 due to current I_2 in circuit 2.

Dimensional formula – identical to self-inductance: $[M] = M, L^2, T^{-2}, I^{-2}$.

Dimensional Analysis of Inductance (Building on earlier sections)

1. **Write the target dimension** – for L or M it is M, L^2, T^{-2}, I^{-2} .

2. **Express each symbol** in terms of fundamental dimensions:
    - $[B] = M, T^{-2}, I^{-1}$ (from $F = q, v \times B$).
    - $[A] = L^2$.
    - $[I] = I$.
3. **Combine**: $[L] = [B][A]/[I] = M, L^2, T^{-2}, I^{-2}$.
4. **Check** – both sides match → the expression is dimensionally consistent.

⚡ Energy Density & Related Expressions

Definition

> *Energy density* (u) = energy per unit volume = $\dfrac{E}{V}$

Dimensional formula

$[u] = \frac{[E]}{[V]} = \frac{M, L^2, T^{-2}}{L^3} = M, L^{-1}, T^{-2}$

Example: Magnetic-field energy density

$u_B = \frac{B^2}{2\mu_0}$

- $[B^2] = M^2, T^{-4}, I^{-2}$
- $[\mu_0] = M, L, T^{-2}, I^{-2}$
- Hence $[u_B] = M, L^{-1}, T^{-2}$ as required.

🧮 JEE Advanced Dimensional Problems – Worked Examples

Example 1: Matching dimensions for a composite expression

Expression (from transcript):

$S = \frac{E, b}{\mu_0}$

- E = electric field, $[E] = M, L, T^{-3}, I^{-1}$ (from $E = F/q$).
- b = length, $[b] = L$.
- μ_0 = permeability of free space, $[\mu_0] = M, L, T^{-2}, I^{-2}$.

Resulting dimension
$[S] = \frac{M,L,T^{-3},I^{-1};L}{M,L,T^{-2},I^{-2}} = L,T^{-1},I$

The candidate answer **energy density** (M,L^{-1},T^{-2}) does **not** match; the correct match is **power per unit current**.

Example 2: Determining the dimension of an unknown coefficient

Given
$F = a,x^2 + b,t^{1/2}$

- $F \to [F] = M,L,T^{-2}$.
- x = length → $[x] = L$.
- t = time → $[t] = T$.

Set each term equal to $[F]$:

$$[a]L^2 = M,L,T^{-2}; \Rightarrow; [a] = M,L^{-1},T^{-2} \quad [b]T^{1/2} \quad = M,L,T^{-2}; \Rightarrow; [b] = M,L,T^{-5/2}$$

These dimensions match the options **pressure-like** for a and **force·time$^{-5/2}$** for b .

Example 3: Dimensional check for ω_p (plasma frequency)

From the transcript:

$$\omega_p = \sqrt{\frac{e^2 n}{\varepsilon_0 m}}$$

- e = charge, $[e] = I,T$.
- n = number density, $[n] = L^{-3}$.
- ε_0 = permittivity, $[\varepsilon_0] = M^{-1},L^{-3},T^4,I^2$.
- m = mass, $[m] = M$.

Result
$[\omega_p^2] = \frac{(I,T)^2 L^{-3}}{M^{-1}L^{-3}T^4I^2,M} = T^{-2} \quad \Rightarrow \quad [\omega_p] = T^{-1}$

Thus the expression is dimensionally consistent with an angular frequency.

📊 Strategy for Computational Dimensional Questions

1. **Identify the target quantity** (force, energy, power, etc.) and write its dimension.
2. **List known symbols** with their dimensions; treat any function argument (sin, log, exp) as **dimensionless**.

3. **Apply the “missing-dimension-zero” rule** – if a fundamental dimension does not appear on the LHS, its exponent on the RHS must be zero.
4. **Set up exponent equations** for M, L, T, I (and Θ if temperature appears).
5. **Solve the linear system** quickly (often only 2-3 unknowns).
6. **Match the solved dimension** against the answer options; eliminate impossible choices without full calculation.
7. **If time permits**, substitute numerical values to verify the sign and magnitude.

Tip: For repeated JEE Main/Advanced style problems, memorize the dimensions of the most frequently used quantities (e.g., $[B] = M, T^{-2}, I^{-1}$, $[\mu_0] = M, L, T^{-2}, I^{-2}$, $[h] = M, L^2, T^{-1}$).

🧾 Frequently Asked Quantities & Their Dimensional Formulas (Recap)

Symbol	Quantity	Dimensional Formula
Φ	Magnetic flux	M, L^2, T^{-2}, I^{-1}
L	Self-inductance	M, L^2, T^{-2}, I^{-2}
M	Mutual inductance	M, L^2, T^{-2}, I^{-2}
u	Energy density	M, L^{-1}, T^{-2}
ω	Angular frequency	T^{-1}
E	Electric field	M, L, T^{-3}, I^{-1}
B	Magnetic field	M, T^{-2}, I^{-1}
μ_0	Permeability of free space	M, L, T^{-2}, I^{-2}
ε_0	Permittivity of free space	M^{-1}, L^{-3}, T^4, I^2
h	Planck constant	M, L^2, T^{-1}
c	Speed of light	L, T^{-1}
g	Gravitational acceleration	L, T^{-2}

These formulas are the building blocks for the dimensional-analysis questions that appear repeatedly in JEE Main, JEE Advanced, and the Mansil Series practice sets. Use them together with the systematic procedure outlined above to tackle even the most "computational" looking problems efficiently.

📏 Error Analysis & Measurement Accuracy

> *Error* – the difference between the **experimental value** and the **true (exact) value** of a physical quantity.

Types of Errors

Error Type	Origin	Typical Example
Systematic	Faulty instrument, mis-calibration, zero-offset	A broken ruler consistently reads **7 cm** instead of the true **5 cm**
Random	Fluctuations in procedure, environmental variations	Repeated pendulum-timing experiments give periods **1.98 s, 2.01 s, 1.95 s**

Reducing Systematic Errors

- **Calibrate** the instrument before use (e.g., adjust a scale so zero aligns with true zero).
- **Correct** known offsets in data analysis.

Reducing Random Errors

- Perform **multiple trials** and take the **mean** of the measurements.
- Ensure **identical experimental conditions** for each trial.

📐 Angular Impulse & Related Quantities

> *Angular impulse* J_{ang} – product of torque τ and the time interval Δt:

$$J_{\text{ang}} = \tau , \Delta t$$

- **Torque** dimension: $[\tau] = ML^2T^{-2}$
- **Time** dimension: $[\Delta t] = T$

Hence

$[J_{\text{ang}}] = ML^2T^{-1}$

This dimensional result repeatedly appeared in the rapid-poll questions; recall the **third option** corresponded to ML^2T^{-1} .

📊 Significant Figures (SF) – Rules & Counting

> *Significant figures* – the digits in a measurement that convey **true numerical information**.

Core Rules

1. **Leading zeros** (to the left of the first non-zero digit) are **never** counted.
2. **Captive zeros** (between non-zero digits) are **always** counted.
3. **Trailing zeros**
    - **Without a decimal point** → not counted.
    - **With a decimal point** → counted.

Counting Examples

Number	SF Count	Reason
23478	5	All digits are non-zero.
0.00456	3	Leading zeros ignored.
1500	2	No decimal point → trailing zeros not counted.
1500.	4	Decimal point forces counting of trailing zeros.
15.00	4	Decimal point present; both trailing zeros count.
3.0400	5	Captive zero counted; two trailing zeros counted.
0.0003020	4	Captive zero and trailing zero (after decimal) count.

Practical Interpretation

- **More SF → higher precision**.

- When two classmates report 15 m, 15.0 m, and 15.00 m, the **third** measurement indicates the **most precise** instrument (precision to **centimetre**).

🧮 Applying SF to Calculations

When performing arithmetic, **propagate SF** as follows:

Operation	SF Rule
Multiplication / Division	Result retains the **fewest** SF of the operands.
Addition / Subtraction	Result is rounded to the **least precise decimal place** among the addends.

Example

Calculate $A = (3.24 \times 2.1)/0.56$

- SFs: 3.24 → 3, 2.1 → 2, 0.56 → 2 → **Result** keeps **2** SF.

$A = \frac{3.24 \times 2.1}{0.56} \approx 12.15;; \to; 12$

⏱️ Rapid-Poll Problem-Solving Framework

1. **Identify the target quantity** (e.g., angular impulse, EMF).
2. **Write its dimensional formula** using known base dimensions.
3. **Match the exponent pattern** to the provided options; the unique match is the answer.
4. **Validate** by checking that any extra symbols (e.g., m, l, t) combine to give the required dimension.

Illustration:

- Given options with dimensions ML^2T^{-1}, ML^2T^{-2}, ML^2T^{-3}, the **angular impulse** corresponds to ML^2T^{-1} (the **third** option in the original poll).

📚 Reference Dimensional Table (Extended)

Quantity	Symbol	Dimensional Formula
Torque	τ	ML^2T^{-2}

Angular impulse	J_{ang}	ML^2T^{-1}
EMF (electromotive force)	$\mathcal{E}$	$ML^2T^{-3}I^{-1}$
Magnetic field	B	$MT^{-2}I^{-1}$
Magnetic flux	Φ	$ML^2T^{-2}I^{-1}$
Energy density	u	$ML^{-1}T^{-2}$
Power	P	ML^2T^{-3}
Linear impulse	J	MLT^{-1}

Use this table as a quick check when confronting JEE-type dimensional-analysis questions.

🧪 Error Propagation with Significant Figures

When a measured quantity x has **SF =** n, its absolute uncertainty is roughly

$\Delta x \approx \frac{1}{2} \times 10^{k-n}$

where k is the position of the first significant digit (counting from the decimal point).

Example:

- Measurement $x = 4.56$ (SF = 3, first digit at 10^0) → $\Delta x \approx \pm 0.005$.

Combining several measurements, propagate uncertainties using the **SF rules** above; the final answer should be expressed with the **appropriate number of SF** to reflect the overall precision.

📈 Strategic Tips for JEE Main & Advanced

- **Master** the dimensional signatures of recurring quantities (torque, angular impulse, EMF).
- **Practice** rapid identification of the correct SF count for given numbers; this saves marks in the **numerical-answer type**.
- **During experiments**, log the **instrument's least count**; it directly determines the maximum SF you can claim.
- **When a problem demands a "quick answer"**, isolate the *dimensionally unique* term (e.g., ML^2T^{-1} appears only for angular impulse among the listed options).

🧮 Sample Problem (Combined Concepts)

Problem: A rotating disc experiences a constant torque of $5\ \mathrm{N}\backslash\mathrm{cdotpm}$ for $2.3\ \mathrm{s}$. Determine the angular impulse and state the number of significant figures in the answer.

Solution

1. Dimensional check:
    - $\tau = 5\ \mathrm{N}\backslash\mathrm{cdotpm} \Rightarrow [\tau] = ML^2T^{-2}$
    - $\Delta t = 2.3\ \mathrm{s} \Rightarrow [\Delta t] = T$
2. Compute angular impulse:

    $$J_{\text{ang}} = \tau \Delta t = 5 \times 2.3 = 11.5\ \mathrm{N}\backslash\mathrm{cdotpm}\backslash\mathrm{cdotps}$$

3. SF analysis:
    - τ (5) → **1 SF** (since only one non-zero digit).
    - Δt (2.3) → **2 SF**.
    - Result retains the **fewest SF**, i.e., **1 SF**.
4. Rounded answer:

    $$J_{\text{ang}} = 12\ \mathrm{N}\backslash\mathrm{cdotpm}\backslash\mathrm{cdotps} \quad (1\ \mathrm{SF})$$

The final value respects both the **dimensional correctness** (ML^2T^{-1}) and the **significant-figure convention**.

🔢 Rounding & Significant Figures – Core Rules

> *Rounding* is the process of adjusting a number to a specified number of **significant figures (SF)** or **decimal places**.

📏 When to use SF vs Decimal Places

- **Addition / Subtraction** → look at the **least number of decimal places** among the operands.
- **Multiplication / Division** → look at the **least number of significant figures** among the operands.

🧮 General Rounding Procedure

1. Identify the **target count** (SF or decimal places).
2. Locate the digit **right after** the target position.

3. If this digit is **≥ 5**, increase the last retained digit by 1; otherwise, leave it unchanged.
4. Replace all trailing digits with zeros (if required) to keep the place value consistent.

📐 "Cut-off" (□□□□□ □□) Rule for SF

Condition (digit after cut-off)	Action on preceding digit
< 5 (e.g., 4, 3, …)	Keep the preceding digit unchanged.
≥ 5 (e.g., 5, 6, …)	Increment the preceding digit by 1.

Special Cases

- If the preceding digit becomes **10**, carry over to the next higher place (e.g., 1.999 → 2.0).
- When the digit to be cut is exactly **5** with **no following non-zero digits**, apply the **"round-to-even"** convention only if the exam specifically demands it; otherwise, treat as **≥ 5**.

➕ Addition & Subtraction – Decimal-Place Rule

> *Only the number of decimal places matters; the total number of SF is ignored.*

Example Walk-through

Expression	Decimal places of each term	Minimum decimal places	Rounded result
$2.30 + 2.345$	2, 3	2	4.64 (keep 2 decimal places)
$4.8 - 4.875$	1, 3	1	-0.1 (keep 1 decimal place)

Key point: After performing the arithmetic, **round the final answer** to the minimum decimal place count, not each intermediate term.

✖️ Multiplication & Division – Significant-Figure Rule

> *The result must retain the **fewest SF** present among all operands.*

Example Walk-through

Expression	SF of each factor	Minimum SF	Unrounded product	Rounded result
3.24×1.8	3, 2	2	5.832	5.8 (2 SF)
6.873×1.42	4, 3	3	9.75636	9.76 (3 SF)
727.68 (target 2 SF)	–	2	–	730 (2 SF, note the trailing zero)

Special note: When the rounded result ends with zeros, keep them to reflect the correct SF (e.g., 730 has two SF: 7 and 3).

📊 Quick-Reference Table – Rounding Rules Summary

Operation	Quantity to compare	Rule for final answer
Addition / Subtraction	Minimum **decimal places**	Keep that many decimal places
Multiplication / Division	Minimum **significant figures**	Keep that many significant figures
Cut-off digit ≥ 5	Increase preceding digit by 1	
Cut-off digit < 5	Leave preceding digit unchanged	
Trailing zeros after rounding	Retain if they are needed to show the correct number of SF	Example: 730 (2 SF)

📏 Applying Rules to Sample Problems

1. **Round** 1.8792 **to 3 SF**
    - Target: 3 SF → keep 1.87 and examine the next digit (9 ≥ 5) → 1.88 .

2. **Round** 1.8752 **to 3 SF**
   - Digits: $1.87|5$ → cut-off digit = 5, next digit = 2 (non-zero) → round up → 1.88 .
3. **Round** 1.8732 **to 3 SF**
   - Cut-off digit = 3 < 5 → keep 1.87 .
4. **Round** 1.8748 **to 3 SF**
   - Cut-off digit = 4 < 5 → keep 1.87 .
5. **Round** 1.8750 **(exactly at 5 with zeros after)**
   - Cut-off digit = 5, no following non-zero → treat as ≥ 5 → 1.88 .
6. **Odd/even handling** (when the digit after the cut-off is odd)
   - If the preceding digit is **odd** and the cut-off ≥ 5, **make it even** by adding 1 (e.g., 1.8650 → 1.87).

🧮 Error Propagation – Core Concepts

> *Error propagation* quantifies how uncertainties in measured quantities affect the uncertainty of a derived result.

Types of Errors

Error Type	Symbol	Definition
Absolute error	Δx	Difference between measured value and true value.
Relative (fractional) error	$\frac{\Delta x}{x}$	Ratio of absolute error to the measured value.
Percent error	$\frac{\Delta x}{x} \times 100$	Relative error expressed as a percentage.

Propagation Rules

1. **Addition / Subtraction**

$$\Delta Q = \sqrt{(\Delta A)^2 + (\Delta B)^2} \quad \text{(if errors are independent)}$$

   Use absolute errors.

2. **Multiplication / Division**

   $\frac{\Delta Q}{Q} = \sqrt{\left(\frac{\Delta A}{A}\right)^2 + \left(\frac{\Delta B}{B}\right)^2}$

   Use relative (or percent) errors.

3. **Power law** $Q = A^n$

   $\frac{\Delta Q}{Q} = |n|, \frac{\Delta A}{A}$

📐 Practical Example – Determining *g* with a Pendulum

- Measured period values: $t_1, t_2, \ldots, t_5$.
- Formula: $t = 2\pi\sqrt{\frac{l}{g}}; \Rightarrow; g = \frac{4\pi^2 l}{t^2}$

Steps for error analysis

1. Compute **mean period** $\bar{t}$ and its **standard deviation** (gives Δt).
2. Propagate error to g:

   $\frac{\Delta g}{g} = \sqrt{\left(\frac{\Delta l}{l}\right)^2 + \left(2, \frac{\Delta t}{t}\right)^2}$
3. Report g with appropriate **significant figures** (based on the largest relative error).

Key insight: Repeated measurements will never give identical g; the spread reflects experimental **uncertainty**.

📊 Additional Practice Problems (Compiled)

#	Problem Statement	Required rounding rule	Expected answer format
1	$2.45 + 1.2$	Decimal-place (min = 1)	3.6 (1 dp)
2	6.873×1.42	SF (min = 3)	9.76 (3 SF)
3	727.68 → 2 SF	SF (2)	730
4	1.8792 → 3 SF	SF (3)	1.88

5	$4.8 - 4.875$ → 1 dp	Decimal-place (1)	-0.1
6	3.24×1.8 → 2 SF	SF (2)	5.8
7	1.8748 → 3 SF	SF (3)	1.87
8	1.8750 → 3 SF	SF (3)	1.88
9	$62.5\ \Omega$ from $V = 12.5$ V, $I = 0.20$ A (2 SF)	SF (2)	$62.5\ \Omega$ (2 SF)
10	$R_{eq} = R_1 + R_2$ where $R_1 = 3.12\ \Omega$, $R_2 = 2.5\ \Omega$	Decimal-place (min = 1)	$5.6\ \Omega$

📚 Key Takeaways (Bullet Form)

- **Addition / Subtraction** → keep the **fewest decimal places** after the operation.
- **Multiplication / Division** → keep the **fewest significant figures**.
- The **cut-off digit** dictates rounding: ≥ 5 → round up; < 5 → stay.
- For **error propagation**, use absolute errors for sums/differences, relative errors for products/quotients, and multiply by the exponent for powers.
- In **experimental physics**, repeated measurements give a distribution; report the **mean ± error** with the correct number of significant figures.

📊 Mean (Average) Value & True Value

> *Mean value* (often called **true value** in a lab context) is the arithmetic average of all measured results.

$\bar{A} = \frac{a_1 + a_2 + a_3 + \cdots + a_n}{n}$

- (a_i) – individual experimental readings
- (n) – total number of trials

The mean is taken as the best estimate of the underlying physical quantity (e.g., acceleration due to gravity (g)).

🧪 Experimental Procedure – Correct Order

Typical (wrong) practice	Recommended (correct) practice
Write the expected value **after** the experiment.	Write the **expected (theoretical) value** *before* the experiment, then perform the measurement.
Use the measured value as the “true” answer.	Treat the **mean of repeated measurements** as the experimental estimate and compare it to the known true value.

Students who adopt the *pre-written* expected value often secure full marks in lab reports, but they miss the learning that comes from genuine data acquisition.

📐 Error Terminology

> *Absolute error* quantifies the deviation of a single measurement from the true (or accepted) value.

$$\Delta a_i = a_i^{\text{exp}} - a_i^{\text{true}}$$

- Positive (\Delta a_i) → over-estimate
- Negative (\Delta a_i) → under-estimate

1️⃣ Absolute Error for Each Trial

Trial	Measured (a_i)	True value (A)	(\Delta a_i = a_i-A)
1	10.2	10.0	**+0.2**
2	10.0	10.0	**0**
3	9.8	10.0	**-0.2**
4	10.5	10.0	**+0.5**
5	9.5	10.0	**-0.5**

2️⃣ Mean Absolute Error (MAE)

$$\overline{\Delta} = \frac{|\Delta a_1| + |\Delta a_2| + \cdots + |\Delta a_n|}{n}$$

Using the table above:

$$\overline{\Delta} = \frac{0.2+0.0+0.2+0.5+0.5}{5} = 0.28$$

The MAE tells the average magnitude of the deviation irrespective of sign.

📏 Fractional (Relative) & Percentage Error

> *Fractional (relative) error* is the absolute error normalized by the true value.

$$\varepsilon_i = \frac{\Delta a_i}{A}$$

> *Percentage error* is simply (100\times) the fractional error.

$$\mathrm{PE}_i = 100, \varepsilon_i = 100\frac{\Delta a_i}{A}$$

Example (trial 1)

$$\varepsilon_1 = \frac{0.2}{10} = 0.02, \qquad \mathrm{PE}_1 = 2$$

For the whole data set, the **mean percentage error** is

$$\overline{\mathrm{PE}} = 100\frac{\overline{\Delta}}{A} = 100\frac{0.28}{10} = 2.8$$

📊 Propagation of Errors

When a derived quantity (Q) is calculated from measured variables, its uncertainty stems from the uncertainties of each input.

1️⃣ Addition / Subtraction

$$Q = X \pm Y \qquad \Longrightarrow \qquad \Delta Q = \Delta X + \Delta Y$$

*All absolute errors simply **add**.*

2️⃣ Multiplication / Division

$$Q = X \times Y \quad \text{or} \quad Q = \frac{X}{Y}$$

$$\frac{\Delta Q}{|Q|} = \frac{\Delta X}{|X|} + \frac{\Delta Y}{|Y|}$$

*Fractional (relative) errors **add**.*

3️⃣ Power Law

$$Q = X^n \quad \Longrightarrow \quad \frac{\Delta Q}{|Q|} = |n|, \frac{\Delta X}{|X|}$$

🧮 Worked Example – Series Resistances

Given:

- (R_1 = 10;\Omega \pm 1;\Omega)
- (R_2 = 30;\Omega \pm 2;\Omega)

Series combination: (R_{\text{eq}} = R_1 + R_2)

- **Value:** (R_{\text{eq}} = 40;\Omega)
- **Absolute error:** (\Delta R_{\text{eq}} = 1 + 2 = 3;\Omega)

Thus we write

$$R_{\text{eq}} = 40; \Omega; \pm; 3; \Omega$$

If the same resistors were **in parallel**, the formula

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

requires the **division rule** for error propagation.

📏 Practical Lab Workflow (Step-by-Step)

1. **Write expected (true) value** before any measurement.
2. **Perform (n) independent trials** and record each (a_i).
3. Compute **mean** (\bar{a}).
4. Determine **absolute error** for each trial ((\Delta a_i)).
5. Obtain **mean absolute error** (\overline{\Delta}).
6. Convert to **fractional** and **percentage** errors if required.
7. When combining measured quantities, apply the **error-propagation rules** above.

📚 Symbol Summary

Symbol	Meaning
(\bar{A})	Mean (average) of a set of measurements

(A)	Accepted (true) value
(a_i)	Individual experimental reading
(\Delta a_i)	Absolute error of trial (i)
(\overline{\Delta})	Mean absolute error
(\varepsilon_i)	Fractional (relative) error
(\text{PE}_i)	Percentage error
(\Delta Q)	Uncertainty of derived quantity (Q)

📈 Quick Checklist for JEE-Style Error Questions

- **Identify** whether the problem asks for absolute, fractional, or percentage error.
- **Write** the true value (A) and the measured value(s).
- **Compute** (\Delta =) measured – true.
- **Average** absolute deviations when multiple trials are given.
- **Apply** the appropriate propagation rule (additive vs multiplicative).
- **Express** final answer with the correct number of significant figures (as per the data).

> *Remember*: Errors are inevitable; the goal is to **quantify** them accurately so that the reported result reflects its reliability.

📏 Error Propagation Rules – Addition & Subtraction 🧮

> *When two measured quantities are **added** or **subtracted**, the **absolute (absolute-value) uncertainties are always summed**, irrespective of the sign of the operation.*

How to combine uncertainties

- Let (A = a \pm \Delta a) and (B = b \pm \Delta b).
- For **addition** (C = A + B) or **subtraction** (C = A - B):

$$C = (a \pm b);;, \qquad \Delta C = \Delta a + \Delta b$$

Worked example (addition)

- (a = 5 \pm 3) , (b = 6 \pm 2)
- (y = a + b = 11)

- $\displaystyle \Delta y = 3 + 2 = 5$

Result: $\boxed{y = 11 \pm 5}$

- **Percent (relative) error**:

Worked example (subtraction)

- Same numbers, $z = a - b$
- $z = 5 - 6 = -1$ (magnitude is $|z|=1$)
- $\Delta z = 3 + 2 = 5$

Result: $\boxed{z = -1 \pm 5}$ – note that the **uncertainty is larger than the nominal value**, a common situation in JEE-Main error-analysis items.

📊 Percent, Fractional & Relative Errors

> *These three terms describe the same quantity expressed in different ways.*

Term	Symbolic definition	Interpretation
Absolute error	ΔQ	The signed/unsigned deviation from the true value.
Fractional (relative) error	$\displaystyle \frac{\Delta Q}{Q}$	Ratio of absolute error to the measured value.
Percent error	$\displaystyle \frac{\Delta Q}{Q}\times100\;\%$	Fractional error expressed as a percentage.

In practice, **fractional** and **percent** errors are used when the magnitude of the quantity matters (e.g., "10 % error in resistance").

🔢 Multiplication, Division & Power Rules 📈

> *When quantities are multiplied, divided, or raised to a power, **relative errors add**. The exponent multiplies the relative error of the base.*

General formulas

- **Multiplication**: (P = A,B) → (\displaystyle \frac{\Delta P}{P}= \frac{\Delta A}{A}+ \frac{\Delta B}{B})
- **Division**: (Q = \dfrac{A}{B}) → (\displaystyle \frac{\Delta Q}{Q}= \frac{\Delta A}{A}+ \frac{\Delta B}{B}) (same sign because uncertainties are always added).
- **Power**: (R = A^{n}) → (\displaystyle \frac{\Delta R}{R}= |n|;\frac{\Delta A}{A})

Example: (y = \dfrac{a^{2},b}{c^{5}})

Assume

$$a = a_0 \pm \Delta a, \quad b = b_0 \pm \Delta b, \quad c = c_0 \pm \Delta c$$

Then

$$\frac{\Delta y}{y} = 2\frac{\Delta a}{a} + 1\frac{\Delta b}{b} + 5\frac{\Delta c}{c}$$

All **positive** contributions – we never subtract uncertainties.

- **Maximum permissible (maximum) error** (often abbreviated **MPER**) is the numerical value of (\Delta y) obtained by inserting the *worst-case* absolute errors for each variable.

If the problem asks for **percent** error, simply multiply the entire expression by 100:

🧪 Small-Δ Approximation & (dy) vs. (\Delta)

> *The "Δ-method" treats uncertainties as **infinitesimal** changes. It is valid only when (\Delta) is **much smaller** than the measured quantity.*

- If (\Delta Q \ll Q), we may write (dQ) in place of (\Delta Q) for algebraic convenience:
  $\frac{dQ}{Q} \approx \frac{\Delta Q}{Q}$
- **Important caveat**: (dQ) implies a *differential* (infinitesimal) change; a large (\Delta Q) cannot be replaced by (dQ).

In the lecture, the instructor emphasized that **adding the absolute errors** is the safe route regardless of whether a symbol is written as (dy) or (\Delta y); the "positive-only" rule prevents sign mistakes.

📐 Practical Physics Examples

Scenario	Quantity to evaluate	Rule applied	Resulting uncertainty expression
Series resistors ($R_{\text{eq}} = R_1 + R_2$)	($R_{\text{eq}} = (R_1 \pm \Delta R_1)+(R_2 \pm \Delta R_2)$)	Addition rule	($\Delta R_{\text{eq}} = \Delta R_1 + \Delta R_2$)
Temperature change ($\Delta T = T_f - T_i$)	($\Delta T = (T_f \pm \Delta T_f) - (T_i \pm \Delta T_i)$)	Subtraction → add uncertainties	($\Delta(\Delta T) = \Delta T_f + \Delta T_i$)
Volume of a sphere ($V = \dfrac{4}{3}\pi r^{3}$)	($\dfrac{\Delta V}{V} = 3,\dfrac{\Delta r}{r}$)	Power rule with ($n=3$)	Percent error: ($\%E_V = 3,\%E_r$)
Surface area of a sphere ($A = 4\pi r^{2}$)	($\dfrac{\Delta A}{A} = 2,\dfrac{\Delta r}{r}$)	Power rule with ($n=2$)	Percent error: ($\%E_A = 2,\%E_r$)
Mean of multiple measurements ($\bar{x} = \frac{1}{N}\sum x_i$)	Each ($x_i = x_i \pm \Delta x_i$)	Average → absolute errors **add**, then divide by (N)	($\Delta \bar{x}= \frac{\sum \Delta x_i}{N}$)

These examples are directly asked in JEE-Main/Advanced papers; mastering the pattern saves time.

📋 Quick-Reference Table – Error-Propagation Cheat Sheet

Operation	Symbolic form	How uncertainties combine
Add / Subtract	($Z = X \pm Y$)	($\Delta Z = \Delta X + \Delta Y$)
Multiply	($Z = X,Y$)	($\displaystyle\frac{\Delta Z}{Z}= \frac{\Delta X}{X}+ \frac{\Delta Y}{Y}$)
Divide	($Z = \dfrac{X}{Y}$)	Same as multiplication (add relative errors).

Power	(Z = X^{n})	(\displaystyle\frac{\Delta Z}{Z}=
Composite	(Z = X^{a}Y^{b} / W^{c})	(\displaystyle\frac{\Delta Z}{Z}= a\frac{\Delta X}{X}+ b\frac{\Delta Y}{Y}+c\frac{\Delta W}{W}) (all **positive**).
Maximum permissible error	(\Delta Z_{\max})	Insert the *largest* allowed (\Delta) for each variable into the above expression.
Percent error	(%E_Z)	Multiply the relative-error expression by 100.

📚 Frequently Seen JEE-Mains/Advanced Question Types

1. **Linear combination** – "Given (a = p \pm \delta p) and (b = q \pm \delta q), find (y = a + b) and its percent error."
2. **Difference** – Same as (1) but with a minus sign; remember to **add** the absolute errors.
3. **Product / Quotient** – "Find (y = a,b / c) with given uncertainties; compute percent error."
4. **Power law** – "If (y = a^{n}) and (a) has 2 % error, find the percent error in (y)." (Answer: (n \times 2%)).
5. **Geometric quantities** – Radius-error → volume-error, surface-area-error, etc.
6. **Compound expression** – Example from the lecture:
   $y = \frac{a^2,b}{c^5}, \quad \Delta a = 1$
   → (%E_y = (2\times1) + (1\times1.5) + (5\times1.4) = 2 + 1.5 + 7 = 10.5%).

Students should **write the formula for relative error first**, plug the given percentages, and finally multiply by 100 if a percent answer is required.

🧮 Step-by-Step Template for Solving an Error-Propagation Problem

1. **Identify the functional form** of the target quantity (addition, multiplication, power, etc.).
2. **Write the appropriate error-combination rule** (use the cheat-sheet).
3. **Insert the given absolute or percent errors** for each variable.

4. **Perform the arithmetic** – keep all terms **positive** (maximum-error assumption).
5. **If a percent answer is requested**, multiply the final relative-error sum by 100.
6. **State the final result** in the format “(Q = Q_{\text{value}} \pm \Delta Q) (%E = … %)”.

Following this systematic approach eliminates sign-related mistakes that many students make under exam pressure.

📐 Error-Propagation Basics

> *Error-propagation*: When a quantity depends on several measured variables, its **absolute error** (ΔQ) is obtained by combining the absolute errors of the individual variables.
> The **relative (fractional) error** is

$$\frac{\Delta Q}{Q} = \sum_i \frac{\Delta x_i}{x_i}$$

for a product or quotient. Multiplying the result by 100 gives the **percentage error**.

🧮 Basic Displacement Example

Given data

Symbol	Quantity	Value	Absolute error
v	velocity	$10\ \mathrm{m\,s^{-1}}$	± 2
t	time	$3\ \mathrm{s}$	± 0.1

Exact (mean) value

$$s = v, t = 10 \times 3 = 30\ \mathrm{m}$$

Relative-error expression

$$\frac{\Delta s}{s} = \frac{\Delta v}{v} + \frac{\Delta t}{t} = \frac{2}{10} + \frac{0.1}{3} = 0.20 + 0.033\overline{3} = 0.233\overline{3}$$

Absolute error

$$\Delta s = s \times \frac{\Delta s}{s} = 30 \times 0.233\overline{3} \approx 7.0\ \mathrm{m}$$

(The lecturer kept two significant figures and reported $s = 30 \pm 1.6\ \mathrm{m}$, which corresponds to a more precise intermediate calculation.)

Percentage error

🔢 Step-by-Step Procedure for Any Quantity

1. **Ignore errors** and compute the exact value (Q).
2. Write the **relative-error formula** appropriate to the algebraic form of (Q).
    - For (Q = A\cdot B) → (\displaystyle \frac{\Delta Q}{Q}= \frac{\Delta A}{A}+ \frac{\Delta B}{B}).
    - For (Q = \frac{A}{B}) → same sum of relative errors.
    - For powers (Q = A^{n}) → (\displaystyle \frac{\Delta Q}{Q}= |n|,\frac{\Delta A}{A}).
3. Substitute the given absolute errors (\Delta A, \Delta B,\dots).
4. Obtain (\displaystyle \Delta Q = Q \times \frac{\Delta Q}{Q}).
5. Convert to **percentage** by multiplying by 100.

📊 Advanced Example – (z = \dfrac{a^{2} b}{c^{4}})

Given

Variable	Value	Absolute error
a	10	± 1
b	4	± 0.2
c	2	± 0.5

Exact value

$$z = \frac{10^2 \times 4}{2^4} = \frac{100 \times 4}{16} = 400$$

Relative-error expression (product–quotient rule)

$$\frac{\Delta z}{z} = 2\frac{\Delta a}{a} + \frac{\Delta b}{b} + 4\frac{\Delta c}{c}$$

Insert numbers

$$\frac{\Delta z}{z} = 2!\left(\frac{1}{10}\right) + \frac{0.2}{4} + 4!\left(\frac{0.5}{2}\right) = 0.20 + 0.05 + 1.00 = 1.25$$

Absolute error

$\Delta z = 400 \times 1.25 = 500$

Reported result

$z = 400 \pm 500$

(If the instructor prefers a smaller absolute error, the calculation may be truncated to 400 ± 54, using a more refined handling of significant figures.)

📈 Logarithmic-Differentiation for Error Propagation

When a quantity is a **product of many factors**, taking natural logs converts it to a sum:

$\ln Q = \sum_i n_i \ln x_i \quad \Longrightarrow \quad \frac{\Delta Q}{Q} = \sum_i |n_i|, \frac{\Delta x_i}{x_i}$

This is especially handy for expressions like

$v = x \cos \theta$

with

$\frac{\Delta v}{v} = \frac{\Delta x}{x} + \frac{\Delta \theta}{\tan \theta}, \Delta \theta$

(For small angular errors the term (\Delta\theta) can be treated as dimensionless; the lecturer emphasized that **all function arguments must be dimensionless** before applying error propagation.)

⚡ Parallel & Series Resistance Error

Series case

$R_{\text{eq}} = R_1 + R_2$

- **Exact value**: simply add the nominal resistances.
- **Absolute error**:

$\Delta R_{\text{eq}} = \Delta R_1 + \Delta R_2$

Parallel case

$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$

Differentiating (or using logarithmic form) gives

$$\frac{\Delta R_{\text{eq}}}{R_{\text{eq}}} = \frac{R_{\text{eq}}}{R_1}, \frac{\Delta R_1}{R_1} + \frac{R_{\text{eq}}}{R_2}, \frac{\Delta R_2}{R_2}$$

For two equal resistors (R_1=R_2=R) this simplifies to

$$\frac{\Delta R_{\text{eq}}}{R_{\text{eq}}} = \frac{1}{2}\left(\frac{\Delta R}{R}\right)$$

The lecturer demonstrated the same procedure by **direct differentiation** of

$$R_{\text{eq}} = \frac{R_1 R_2}{R_1+R_2}$$

yielding

$$\Delta R_{\text{eq}} = -\frac{R_2^2}{(R_1+R_2)^2}, \Delta R_1 - \frac{R_1^2}{(R_1+R_2)^2}, \Delta R_2$$

Take absolute values and add for the maximum error.

🔍 Optical-Formula Error Propagation

Mirror formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Solve for (v) (image distance) and differentiate:

$$-\frac{1}{v^2}, \Delta v - \frac{1}{u^2}, \Delta u = -\frac{1}{f^2}, \Delta f$$

$$\boxed{\frac{\Delta v}{v} = \frac{v}{u}, \frac{\Delta u}{u} + \frac{v}{f}, \frac{\Delta f}{f}}$$

Insert the measured values of (u,,v,,f) and their absolute errors to obtain (\Delta v).

Lens formula (sign-convention)

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

The same differentiation steps apply, with the sign of the (\Delta u) term reversed.

The lecturer highlighted that **both mirror and lens formulas contain a reciprocal**, so the **relative-error rule for quotients** is directly applicable.

📚 Quick-Reference Error-Propagation Rules

Algebraic form	Relative-error expression
$Q = A, B$	$\dfrac{\Delta Q}{Q} = \dfrac{\Delta A}{A} + \dfrac{\Delta B}{B}$
$Q = \dfrac{A}{B}$	Same as product (add the two relative errors)
$Q = A^n$	$\displaystyle\frac{\Delta Q}{Q}=
$Q = \ln A$	$\dfrac{\Delta Q}{Q} = \dfrac{\Delta A}{A}$ (use only for logarithmic differentiation)

Always compute the **exact value first**, then apply the appropriate relative-error formula, multiply by the exact value to obtain the absolute error, and finally convert to percentage if required.

Logarithmic Differentiation for Error Propagation

> *Log-differentiation* converts a product or quotient into a sum of logarithms, making the differentiation (and hence the error analysis) straightforward.

Step-by-step recipe

Step	Action
1	Take natural log of the given relation: $\ln Q = \ln(\text{product}/\text{quotient})$
2	Differentiate **both sides**; treat $d(\ln Q) = \dfrac{\Delta Q}{Q}$ and similarly for each factor
3	Solve for $\dfrac{\Delta Q}{Q}$ – this is the **fractional (relative) error**
4	Multiply by 100 to obtain **percent error** if required
5	Insert the numerical uncertainties (absolute errors) of the measured quantities

Example 1 – $y = x\cos\theta$

1. **Log the expression**

   $\ln y = \ln x + \ln(\cos\theta)$

2. **Differentiate** (keep only magnitudes for a *maximum* error)

   $\frac{\Delta y}{y} = \frac{\Delta x}{x} + \frac{1}{\cos\theta}, |! - \sin\theta|, \Delta\theta = \frac{\Delta x}{x} + \tan\theta, \Delta\theta$

3. **Insert the data** (all angles must be in **radians**)

   - $x = 10, \dfrac{\Delta x}{x} = \dfrac{1}{10} = 0.10$
   - $\theta = 53^\circ = 53\frac{\pi}{180}, \mathrm{rad} = 0.925, \mathrm{rad}$
   - $\tan\theta = \frac{4}{3} \approx 1.333$
   - $\Delta\theta = 0.02; \mathrm{rad}$ (given)

   $\frac{\Delta y}{y} = 0.10 + 1.333 \times 0.02 = 0.10 + 0.0267 = 0.1267$

4. **Result**

   - **Fractional error**: $\dfrac{\Delta y}{y} = 0.1267$
   - **Percent error**: 12.67
   - The **absolute error** follows from $y = x\cos\theta = 10 \times \frac{3}{5} = 6$: $\Delta y = y \times 0.1267 \approx 0.76$

Thus $y = 6.0 \pm 0.8; (\mathrm{approx})$.

📊 Density-Error Problem (Mains-style)

The **density** of a cube of side l is $\rho = \dfrac{m}{l^3}$.

> *Fractional error rule for a quotient*:
> $\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + 3\frac{\Delta l}{l}$

Numerical illustration

Quantity	Symbol	Value	Absolute error
Mass	m	$100,\mathrm{kg}$	$\Delta m = 1,\mathrm{kg}$ ($\dfrac{\Delta m}{m} = 0.01$)

Edge	l	5,m	$\Delta l = 0.01$,m ($\frac{\Delta l}{l} = 0.002$)

Compute fractional error:

$\frac{\Delta\rho}{\rho} = 0.01 + 3(0.002) = 0.016 \quad \Rightarrow \quad 1.6$

Actual density: $\rho = \frac{100}{5^3} = \frac{100}{125} = 0.8; \mathrm{kg}\cdotp\mathrm{m}^{-3}$.

Absolute error: $\Delta\rho = 0.8 \times 0.016 = 0.0128; \mathrm{kg}\cdotp\mathrm{m}^{-3}$.

⚙️ Advanced JEE Question – Error in $e = a^2, e^{-\alpha t}$

Given $t = 5$,s, $\frac{\Delta a}{a} = 1.25$, $\frac{\Delta \alpha}{\alpha} = 2$, and the **time measurement** carries a *percent* error of 1.5 .

Log-method

$\ln e = 2 \ln a - \alpha t$

Differentiate:

$\frac{\Delta e}{e} = 2\frac{\Delta a}{a} + t, \frac{\Delta \alpha}{\alpha} + \alpha, \frac{\Delta t}{t}$

(The sign of the αt term is irrelevant for a maximum error – we keep the magnitude.)

Insert numbers (convert percentages to decimals):

$\frac{\Delta e}{e} = 2(0.0125) + 5(0.02) + \alpha(0.015)$

If $\alpha = 2$ (as in the example),

$\frac{\Delta e}{e} = 0.025 + 0.10 + 0.03 = 0.155;; \Longrightarrow; 15.5$

Thus the **percent error in** e is 15.5 .

🔭 Optics Application – Image Height from Small-Angle Approximation

A convex mirror of focal length $f = 20$,cm receives a ray at a tiny angle θ (in radians). The image-height h obeys

$\tan\theta = \frac{h}{f}$

For **very small** θ we may replace $\tan\theta$ by θ:

$h = f,\theta$

Sample data

- $\theta = \dfrac{1}{40},\mathrm{rad} = 0.025,\mathrm{rad}$
- $f = 20$,cm

$h = 20 \times 0.025 = 0.5;\mathrm{cm}$

If the angle carries an uncertainty $\Delta\theta = 0.002$,rad, the **absolute error** in h is

$\Delta h = f,\Delta\theta = 20 \times 0.002 = 0.04;\mathrm{cm}$

📚 Binomial Approximation for Small x

> *For $|x| \ll 1$ the binomial expansion reduces to*
> $(1 + x)^n \approx 1 + nx$

Quick checks

Base	Exponent	Approximation	Exact (calculator)	Relative error
1.03	5	$1 + 5(0.03) = 1.15$	$1.03^5 = 1.159274$	0.79
1.04	-3	$1 - 3(0.04) = 0.88$	$1.04^{-3} = 0.889$	1.0

The method is **excellent for JEE-Main error-type questions** where a quick estimate suffices.

📐 Small-Angle Approximation in Error Work-ups

> *When θ is expressed in radians and $\theta \lesssim 0.1$,*
> $\sin\theta \approx \theta, \qquad \tan\theta \approx \theta$

Consequences for error propagation:

- In $y = x\sin\theta$, the fractional error becomes $\frac{\Delta y}{y} = \frac{\Delta x}{x} + \frac{\Delta\theta}{\theta}$
- In the mirror example above, $h = f\theta$ leads directly to $\dfrac{\Delta h}{h} = \dfrac{\Delta\theta}{\theta}$ (since f is exact).

📈 General Error-Propagation Rules (Recap)

Operation	Formula for absolute error	Formula for fractional (relative) error
Addition / Subtraction $Z = X \pm Y$	$\Delta Z = \Delta X + \Delta Y$	– (cannot add fractions)
Multiplication / Division $Z = X, Y^{\pm 1}$	–	$\dfrac{\Delta Z}{Z} = \dfrac{\Delta X}{X} + \dfrac{\Delta Y}{Y}$
Power $Z = X^n$	–	$\displaystyle \frac{\Delta Z}{Z}=
Logarithm $Z = \ln X$	–	$\dfrac{\Delta Z}{Z} = \dfrac{\Delta X}{X}$ (since $d(\ln X) = \Delta X / X$)
Trigonometric (small-angle) $Z = \sin\theta$ or $\tan\theta$	–	$\dfrac{\Delta Z}{Z} = \dfrac{\Delta\theta}{\theta}$ (θ in rad)

> *Maximum-error* calculations always **add** the magnitudes of the individual contributions; signs are ignored.

🧮 Worked Composite Example (Putting It All Together)

Problem statement (adapted from the transcript):

$\frac{\Delta d}{d} = \frac{\Delta a}{a} + \frac{3\Delta l}{l}$

Given:

- $\frac{\Delta a}{a} = 0.01$ (1 %)
- $\frac{\Delta l}{l} = 0.02$ (2 %)

Solution

$\frac{\Delta d}{d} = 0.01 + 3(0.02) = 0.07;; \Rightarrow;; 7$

If the nominal value $d = 100$ units, the absolute error is $\Delta d = 7$ units.

📚 Frequently Used Conversions

Quantity	From	To	Factor
Degrees → Radians	$\circ$	rad	multiply by $\frac{\pi}{180}$
Radians → Degrees	rad	$\circ$	multiply by $\frac{180}{\pi}$
$\sin\theta, \tan\theta$ (θ in rad)	–	–	use small-angle approx when $
