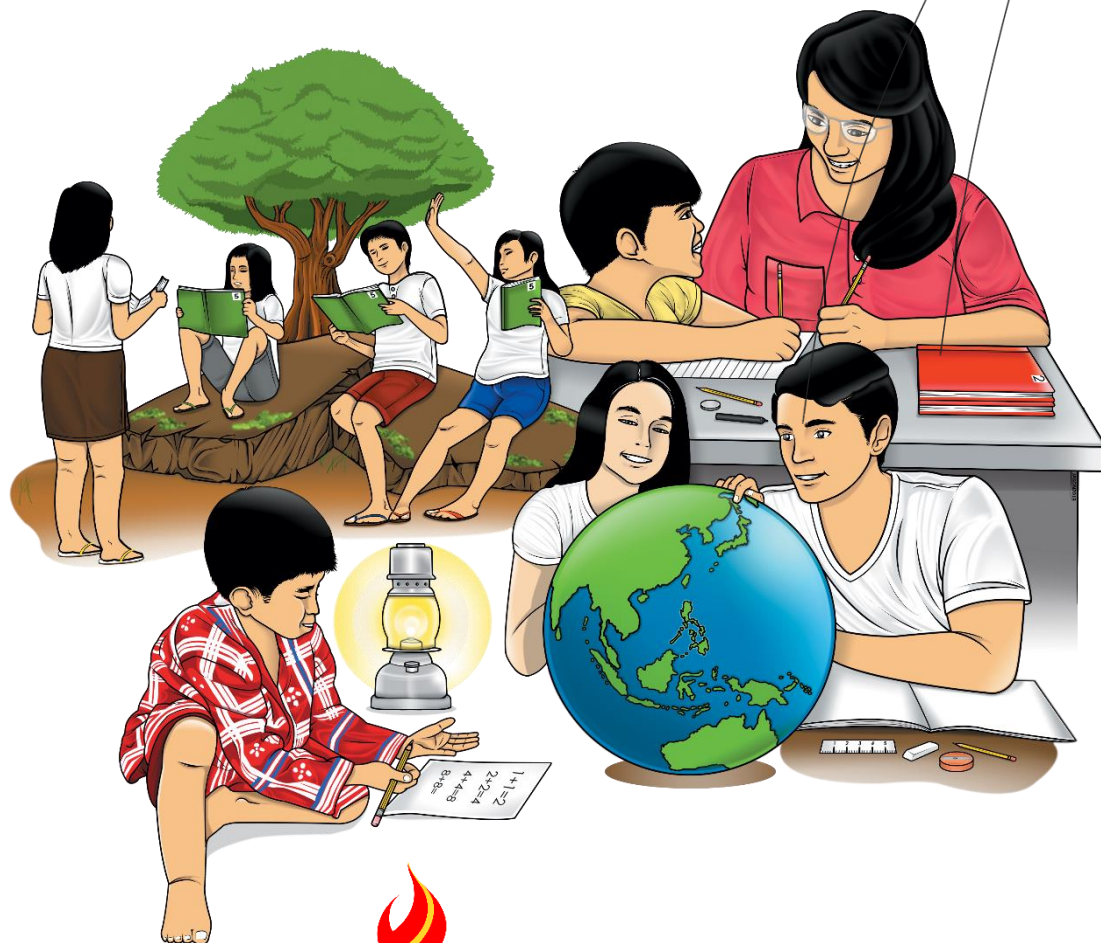


Science

Quarter 4 – Module 2: Height and Range of a Projectile



ALTERNATIVE DELIVERY MODE
ADM

Science – Grade 9

Alternative Delivery Mode

Quarter 4 – Module 2: Height and Range of a Projectile

First Edition, 2020

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Science

Quarter 4 – Module 2: Height and Range of Projectile

Introductory Message

This Self-Learning Module (SLM) is prepared so that you, our dear learners, can continue your studies and learn while at home. Activities, questions, directions, exercises, and discussions are carefully stated for you to understand each lesson.

Each SLM is composed of different parts. Each part shall guide you step-by-step as you discover and understand the lesson prepared for you.

Pre-tests are provided to measure your prior knowledge on lessons in each SLM. This will tell you if you need to proceed on completing this module or if you need to ask your facilitator or your teacher's assistance for better understanding of the lesson. At the end of each module, you need to answer the post-test to self-check your learning. Answer keys are provided for each activity and test. We trust that you will be honest in using these.

In addition to the material in the main text, Notes to the Teacher are also provided to our facilitators and parents for strategies and reminders on how they can best help you on your home-based learning.

Please use this module with care. Do not put unnecessary marks on any part of this SLM. Use a separate sheet of paper in answering the exercises and tests. And read the instructions carefully before performing each task.

If you have any questions in using this SLM or any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator.

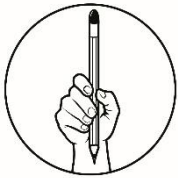
Thank you.



What I Need to Know

In this module, you are expected to investigate the relationship between the angle of release and the height and range of the projectile. To do so, you are also expected to:

1. Describe the relationship between the initial velocity and the height and range of the projectile with constant angle of release
2. Describe the relationship between the angle of release and the height and range of the projectile with constant initial velocity



What I Know

Multiple Choice: Read the following items carefully. On your answer sheet, write the letter of the BEST ANSWER from the given options.

1. At what axis in the cartesian coordinate system is horizontal motion usually associated?
 - a. x-axis
 - b. y-axis
 - c. z-axis
 - d. w-axis
2. What is the acceleration of a freely-falling object on earth?
 - a. 9.8 m/s^2
 - b. -9.8 m/s^2
 - c. 6.4 m/s^2
 - d. Cannot be determined
3. Which of the following statements BEST shows horizontal motion?
 - a. A bird catching its prey
 - b. Skateboarding in a U ramp
 - c. A cheetah leaping to catch a prey
 - d. Running in a 100-yard dash

4. Vertical motion is a type of motion that is affected by gravity. Which of the following satisfies an object moving in vertical motion?
- a. Book on top of a table
 - b. Chandelier suspended in a ceiling
 - c. Ball on the fifth floor of a building
 - d. Fruit falling from a tree
5. What do you call the path followed by a projectile?
- a. Trajectory
 - b. Tracer
 - c. Follow through
 - d. Follow up

Lesson

1

Height and Range of Projectile



What's In

From the previous module, you were introduced to the basic concepts of projectile motion such as trajectory and definition of a projectile itself. A body in projectile motion has been established to have a parabolic trajectory with horizontal and vertical components of position (x and y) and velocity (v_x and v_y).

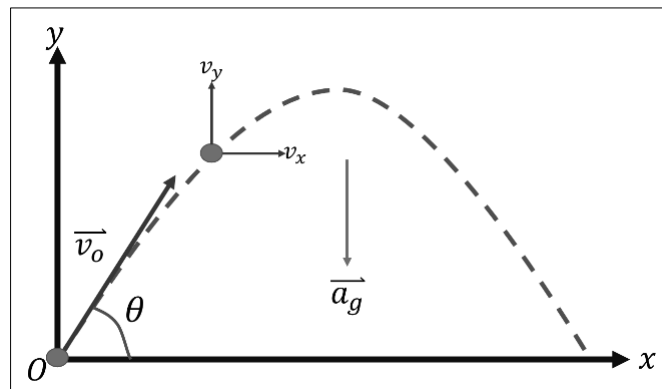


Figure 1. Horizontal and Vertical Components of the trajectory
(Illustrated by S.D. Nijaga)

The projectile motion is the combination of horizontal motion with constant velocity and vertical motion with constant acceleration. The horizontal component of projectile motion (with respect to the x -axis) has an **acceleration equal to zero ($a_x = 0$)** since the horizontal component of the **velocity of projectiles is constant**. On the other hand, the vertical component of acceleration is constant which is equal to the acceleration due to gravity of objects on earth ($a_y = -g$). **Hence, the equations of motion with respect to x and y -axis becomes,**

$$x = x_0 + v_{0x}t \quad \text{and} \quad y = y_0 + v_{0y}t - \frac{1}{2}gt^2, \text{ for the positions}$$

where:

x - final position along x

x_0 - initial position along x

v_{0x} - initial velocity along the x -axis

y - final position along y

y_0 - initial position along x

v_{0y} – initial velocity along the y-axis

t – time

g – acceleration due to gravity (9.81 m/s^2)

and

$$v_x = v_{0x} \text{ and } v_y = v_{0y} - gt, \text{ for the velocities.}$$

where:

v_x – final velocity along the x-axis

v_{0x} – initial velocity along the x-axis

v_y – final velocity along the y-axis

v_{0y} – initial velocity along the y-axis

t – time

g – acceleration due to gravity (9.81 m/s^2)

Let us assume the initial position ($t=0 \text{ s}$) is at the origin, then $x_0=y_0=0$ since this can be the position as a ball leaves a thrower's hand or position of a bullet at the moment it leaves the gun barrel. Since projectile motion is considered as 2-Dimensional motion, an object's motion can be described in two ways, horizontally (how far) or vertically (how high). The height and range an object reaches is dependent upon 1) how fast it leaves its initial position (initial velocity, v_0) and 2) the angle at which it leaves from its initial position (angle of launch, θ) shown in Figure 1.

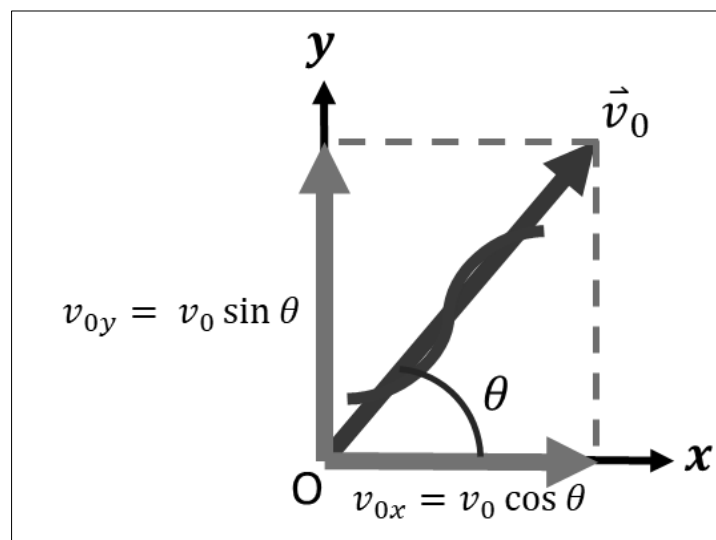


Figure 2. Components of 2-Dimensional Motion in terms of initial velocity and angle of launch. (Illustrated by S.D. Nijaga)

Figure 2 shows the x and y components of the initial velocity with respect to the angle of launch given by the equations,

$$v_{0x} = v_0 \cos \theta \quad \text{and} \quad v_{0y} = v_0 \sin \theta$$

where:

v_{0x} – initial velocity along the x-axis

v_{0y} – initial velocity along the y-axis

t – time

θ - angle between the initial velocity and the x-axis

Hence, the equations of motion become,

$$x = (v_0 \cos \theta)t \quad \text{and} \quad y = (v_0 \sin \theta)t - \frac{1}{2}gt^2 \quad \text{for the positions}$$

and

$$v_x = v_0 \cos \theta \quad \text{and} \quad v_y = v_0 \sin \theta - gt \quad \text{for the velocities.}$$

where:

x – final position along x-axis

y – final position along y-axis

v_0 – initial velocity

v_y – final velocity along y-axis

v_x – final velocity along x-axis

t – time

g – Acceleration due to gravity (9.81 m/s²)



What's New

Analyze the trajectory of the following projectile. The figure shown below is a person shooting a piece of paper into a trash bin. Now, it is your time to do it by preparing a trash bin and a piece of crumpled paper. You may also choose to prepare an alternative to a trash bin if you don't have any available. Write 5 observations about the movement of the crumpled paper after it leaves your hand and before it goes inside the trash bin

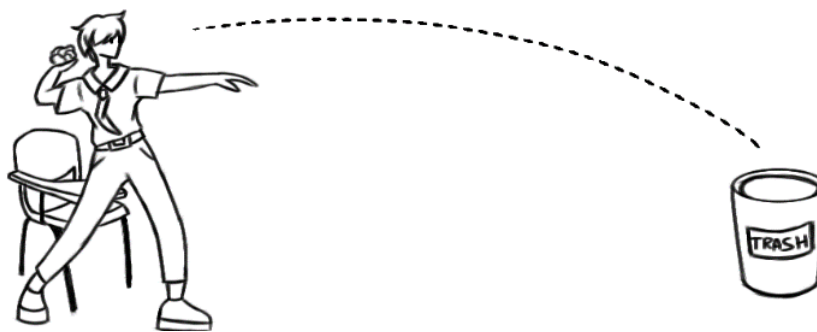


Figure 3. A boy throwing a piece of paper in a trash can (Illustrated by A. Tawano)

OBSERVATIONS

1. _____
2. _____
3. _____
4. _____
5. _____

Rubric

Criteria	4	3	2	1
Amount of Information	5 observations are written	4 Observations are written	3 Observations are written	2 or less observations are written
Relevance	All observations are related to topic	Most Observations are related to topic	Some Observations are related to topic	None of the observation are relevant
Detail of Observation	Well Defined Details of observation	Very Satisfactory Detailed observations	Satisfactory Detailed observations	Unclear observations



What is It

The conditions for projectile motion to happen is that an object is thrown with an initial velocity \vec{v}_0 at an angle of launch θ . Now, let us trace the trajectory of the projectile following a symmetric trajectory (we shall only be dealing with symmetric trajectories for discussion purposes) *by neglecting air resistance*. The body starts at the point origin O.

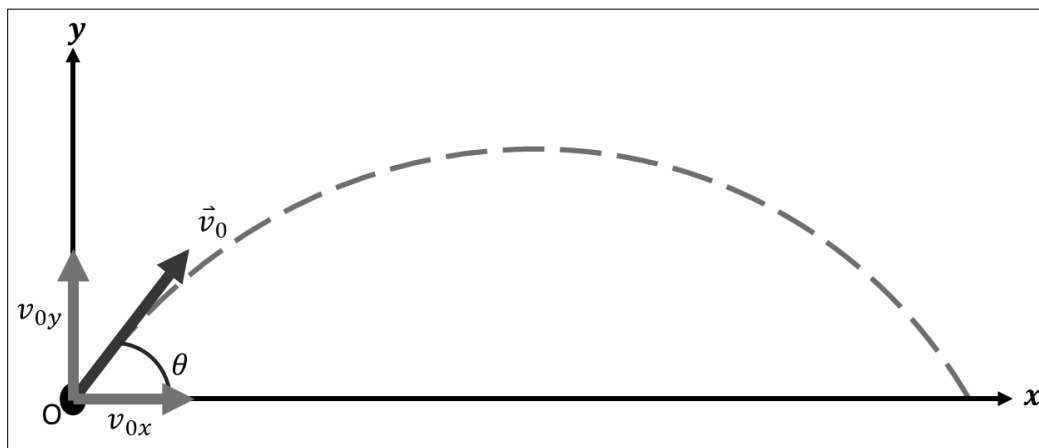


Figure 4. Projectile velocity in x- and y- components (Illustrated by S.D. Nijaga)

As what is shown in the figure, the trajectory of the body is parabolic. The figure above shows the position of the body is **ascending** since the vertical component of velocity v_{0y} is going up. The peak of the parabola is the highest point where the body could be in its trajectory (**$y = H$**). The **peak of the parabola** is where the vertical component of velocity is zero (**$v_y = 0$**) shown in Figure 4.

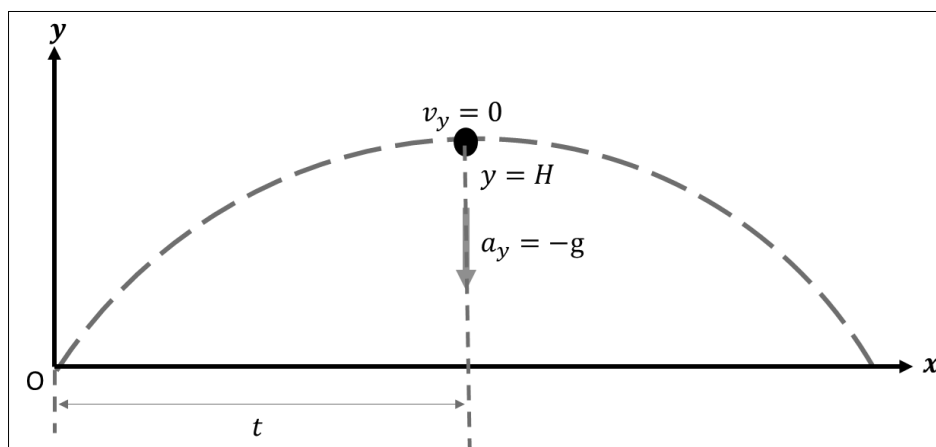


Figure 5. Conditions at maximum height H at time t (Illustrated by S.D. Nijaga)

Applying this to the y-component of the velocity, at the peak of flight of the projectile

$$v_y = v_0 \sin \theta - gt = 0$$

The time when the projectile reached its highest point is then,

$$t = \frac{v_0 \sin \theta}{g}$$

To find the maximum height H, we substitute t to the equation of position along the y-axis. The equation then becomes,

$$\begin{aligned} H &= (v_0 \sin \theta) \frac{v_0 \sin \theta}{g} - \frac{1}{2} g \left(\frac{v_0 \sin \theta}{g} \right)^2 \\ &= \frac{v_0^2 \sin^2 \theta}{g} - \frac{1}{2} \frac{v_0^2 \sin^2 \theta}{g} \\ H &= \frac{v_0^2 \sin^2 \theta}{2g} \end{aligned}$$

The time it takes for a projectile to go down to the same level (t_{down}) is equal to the time it takes when it was rising (t_{up}) to its maximum height,

$$t_{\text{up}} = t_{\text{down}} = t$$

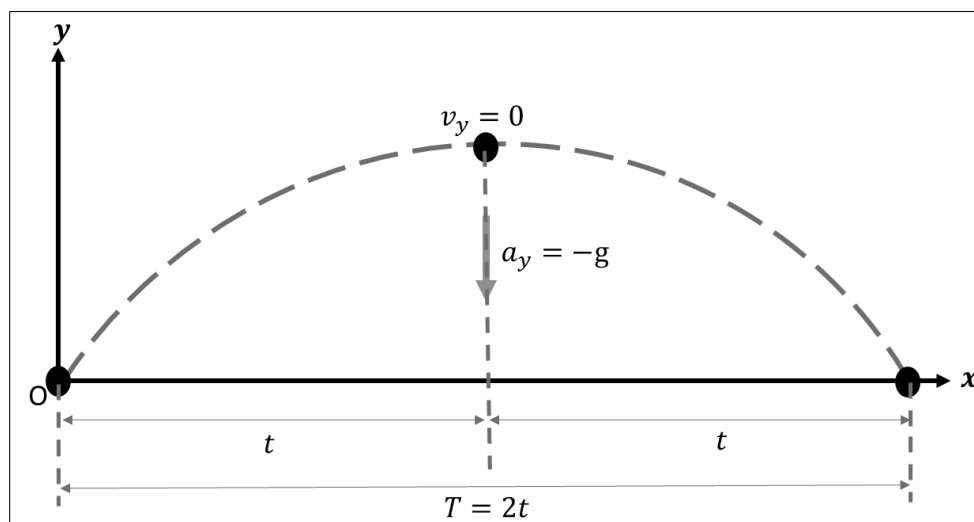


Figure 6. Path of the projectile over time (Illustrated by S.D. Nijaga)

The total time of flight (T) is just double that time, t.

$$T = 2t$$

At the total time of flight, the projectile reaches its maximum range (R) at T shown in Figure 5.

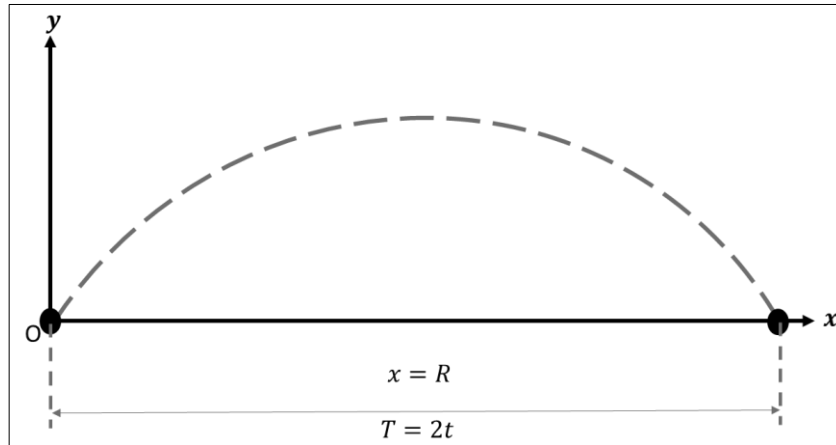


Figure 7. Descending Projectile's trajectory (Illustrated by S.D. Nijaga)

Substituting T to the equation of position along the x-axis, the equation becomes

$$\begin{aligned} R &= (v_0 \cos \theta)T = (v_0 \cos \theta)2t \\ &= (v_0 \cos \theta)2\left(\frac{v_0 \sin \theta}{g}\right) \\ &= \frac{v_0^2 (2 \cos \theta \sin \theta)}{g} \end{aligned}$$

Since $2 \cos \theta \sin \theta = \sin 2\theta$

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

Three important quantities to consider when describing the motion of projectiles are: time, range, and height. The following is a summary of equations to describe these quantities for a projectile

The maximum height is the maximum vertical displacement of projectiles. It is calculated as follows:

$$H = \frac{v_0^2 \sin^2 \theta}{2g}$$

where:

H – Height (horizontal displacement)

v_0 – initial velocity

θ - angle between the horizontal and the direction of the initial velocity

g – acceleration due to gravity (9.81 m/s/s)

The maximum range (R) is the maximum horizontal displacement that can be reached by a projectile. It can be calculated by the following equation:

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

where:

R – Range (vertical displacement)

v_0 – initial velocity

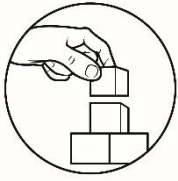
θ - angle between the horizontal and the direction of the initial velocity

g – acceleration due to gravity (9.81 m/s²)

Based from the equations of height and range of a projectile, its values is **dependent upon the angle of launch**.

We can note of the following salient values of the angle theta measured with respect to the horizontal and the resulting range:

- When the angle is zero (0°), it means that the projectile is simply launched along a horizontal displacement ONLY with no vertical displacement. The value of height of a projectile is equal to zero and its range is dependent on the value of the initial velocity and acceleration along the x-axis. (considered as Uniformly Accelerated Motion along x-axis)
- When the angle is 90°, the resulting horizontal displacement is zero, and the vertical displacement along the y-axis only. The value of range of a projectile is equal to zero and its height is dependent on the value of the initial velocity and acceleration along the y-axis (gravity). (considered as Free-Fall)
- Between 1° and 89°, projectile motion is evident given that an initial velocity on the object.
- When the angle is 45°, the resulting horizontal displacement is maximum with high vertical displacement (but not maximum).
- When the angle is 89°, the resulting vertical displacement is maximum with low horizontal displacement (but not minimum).



What's More

Activity 1. Word Search. Circle the 10 words listed below. Words appear straight across, back- word straight across, up and down, down and up, and diagonally.

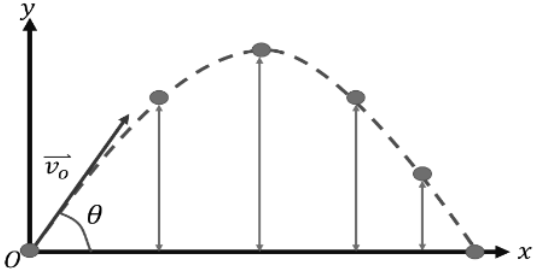
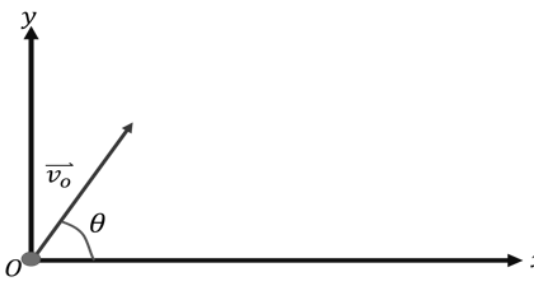
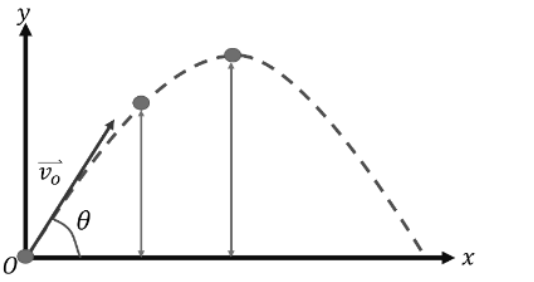
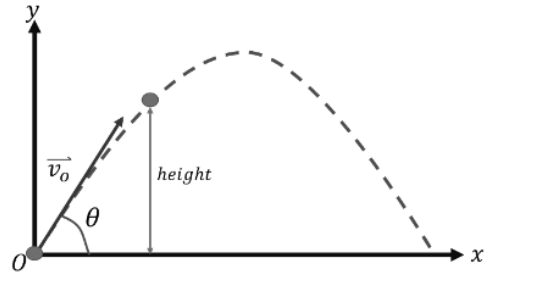
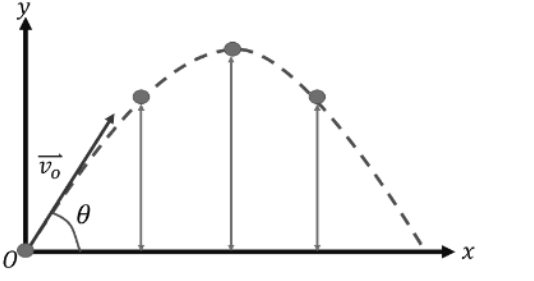
Projectile Motion Word Search

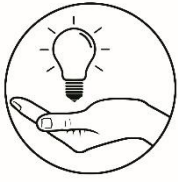
R	C	R	P	C	R	R	R	A	T	H	V	A	I
A	I	R	A	R	T	H	Y	L	O	N	E	L	A
A	A	I	H	V	O	G	O	E	O	E	L	A	N
I	L	T	O	G	J	J	L	H	R	G	O	I	G
N	E	R	R	O	E	I	E	E	O	J	C	R	L
R	I	E	I	A	A	C	G	C	T	T	I	A	E
E	V	T	Z	R	J	N	O	V	T	E	T	Y	E
C	E	E	O	I	A	E	C	I	E	I	Y	V	T
H	R	I	N	R	I	Z	C	L	I	L	L	L	L
E	T	C	T	L	R	A	N	T	A	G	R	E	G
I	I	O	A	T	E	C	I	C	O	U	L	Z	A
G	C	E	L	T	A	L	I	E	R	R	N	Y	C
H	A	E	Y	T	I	V	A	R	G	I	Y	C	L
T	L	J	R	J	I	A	A	O	Y	L	H	T	H

GRAVITY
RANGE
HORIZONTAL
VERTICAL
PROJECTILE
VELOCITY
HEIGHT
LAUNCH
ANGLE
TRAJECTORY

Play this puzzle online at : <https://thewordsearch.com/puzzle/2867284/>

Activity 2: Matching Type. Match the given images of the position of a body in projectile motion in the first column to the example of situations in the second column. Write only the letter of your choice on your answer sheet.

<p>1. </p>	<p>a. Rock at a boy's hand</p>
<p>2. </p>	<p>b. ball on the floor</p>
<p>3. </p>	<p>c. Paper airplane going up</p>
<p>4. </p>	<p>d. Basketball at the top of its flight</p>
<p>5. </p>	<p>e. Person jumping down</p>



What I Have Learned

Fill in the Blanks: There are more examples of objects in projectile motion given below. Supply the best possible answer to the events with regards to projectile motion. Write your answers on a separate sheet of paper.

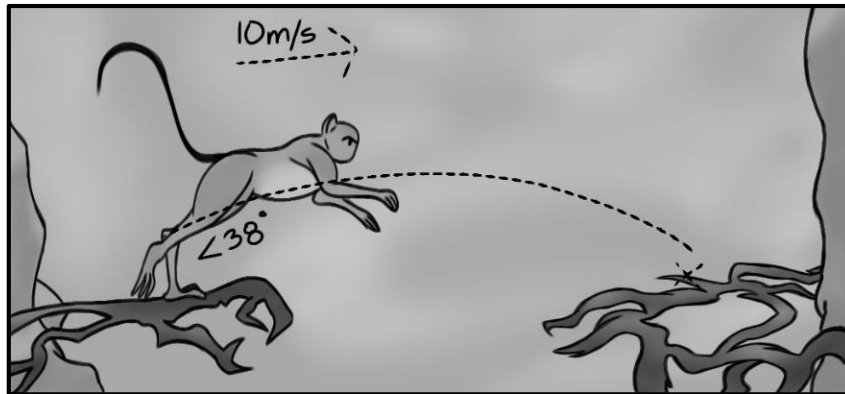


Figure 8. Monkey jumping among branches (Illustrated by S. Robles)

Question: A monkey jumps from one end of the branch with an angle of launch of 38° and initial velocity of 10 m/s. Assume no air resistance (a) How far and high is the monkey at 0.5 s? (b) What is the maximum height of the monkey during its jump? (c) How far should the next branch be before the monkey is unable to reach it?

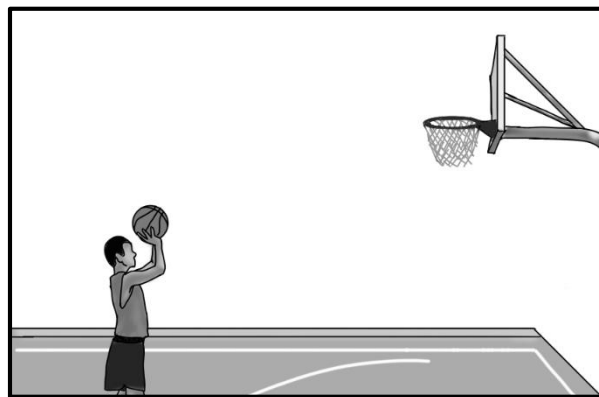


Figure 9. A Basketball player shooting a ball (Illustrated by S. Robles)

Question: The ideal angle of release for most basketball shooters is at 45° . Dirk Nowitski has a release with an angle of 55° . Given an initial velocity of 6 m/s, compare the maximum heights of the shots between the shooters.

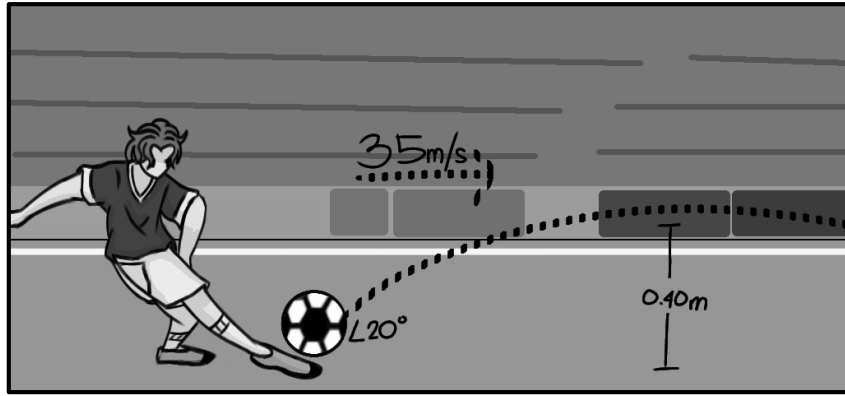
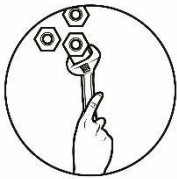


Figure 10. Soccer player kicking the ball (Illustrated by A. Tawano)

Question: The soccer player kicks the ball in the Figure above with an initial velocity of 35 m/s at an angle of 20° . (a) Calculate for the time when it reaches 0.40 m. (b) Find its position parallel to the field at height equal to 0.40 m. (c) Find the time when it reaches its highest point. (d) At what point is the ball at its highest and farthest?



What I Can Do

Answer the following activities.

Activity 1. Write two situations where projectile motion can occur. Describe conditions where the projectile could reach higher maximum height and lowest maximum height. Use another sheet of paper if appropriate.

1. Situation A: _____

Conditions:

2. Situation B: _____

Conditions:

Activity 2. Make a video of Water Rocket Challenge.

Direction. Make a vlog on making a water rocket. Show the water rocket at various heights. Describe how the water rocket reaches its highest possible height.

Rubric for Vlog

Categories	Advanced (10 pts)	Proficient (9 pts)	Developing (7pts)	Beginning (5pts)
Content Quality				
Scientific Concept				
Video Quality				
Sound Quality				
Outline				

Summary

- *Projectile motion is the combination of horizontal motion with constant velocity (acceleration = 0) and vertical motion with constant acceleration (due to gravity).*
- The height and range of a projectile are dependent upon 1) initial velocity and 2) angle of launch
- The trajectory of a projectile is parabolic.
- At the maximum height, the vertical component of the velocity is equal to zero. ($v_y = 0$)
- Before the maximum height, the object is ascending or its height is increasing. After the maximum height, the object is descending or its height is decreasing.
- Given the initial velocity is not changed:
 - The maximum height reached by a projectile increases as the angle of launch increases until an angle of 90 degrees with respect to the horizontal.
 - The maximum height reached by a projectile decreases as the angle of launch decreases from 90 degrees.
 - The range increases as the angle of launch decreases from 45 degrees with respect to the horizontal.
 - Maximum range is attained at an angle of 45 degrees with respect to the horizontal.

- The range decreases as the angle of launch increases from 45 degrees to 90 degrees with respect to the horizontal.
- In most cases, increase in initial velocity implies increase in height and range if angle of launch is constant.



Assessment

Multiple Choice: Read the following items carefully. On your answer sheet, write the letter of the BEST ANSWER from the given options.

1. A stone is just released from the window of a train moving along a horizontal straight track. The stone will hit the ground following:
 - A. Hyperbolic path
 - B. Straight path
 - C. Circular path
 - D. Parabolic path

2. A bullet is dropped from the same height when another bullet is fired horizontally. How will the two bullets reach the ground?
 - A. Simultaneously
 - B. Depends on the observer
 - C. One after the other
 - D. None of the above

3. A bomb is dropped from an airplane moving horizontally at a constant speed. When air resistance is taken into consideration, the bomb _____.
 - A. flies with the airplane
 - B. falls to earth behind the airplane
 - C. falls to earth ahead of the plane
 - D. falls to earth exactly below the airplane

4. A particle moves in a plane with constant acceleration in a direction from the initial velocity. The path of the particle will be _____.
- A. a parabola
 - B. an ellipse
 - C. a straight line
 - D. an arc of a circle
5. A projectile is launched at an angle of 30° above the horizontal. Neglecting air resistance, what are the projectile's horizontal and vertical accelerations when it reaches its maximum height?
- A. Both the horizontal and vertical accelerations are zero.
 - B. The horizontal acceleration is zero and the vertical acceleration is -9.81 m/s^2 .
 - C. The horizontal acceleration is 9.81 m/s^2 and the vertical acceleration is zero.
 - D. Both the horizontal and vertical accelerations are 9.81 m/s^2 .
6. A ball is thrown at angle of 38° with respect to the horizontal. What happens to the magnitude of the ball's vertical velocity during the time interval that the ball is in the air?
- A. It decreases, then increases.
 - B. It decreases, then remains the same.
 - C. It increases, then decreases.
 - D. It remains the same.
7. A baseball player throws a ball horizontally (at an angle of 0°). Which statement best describes the ball's motion after it is thrown? (Neglect air resistance)
- A. Its vertical speed remains the same, and its horizontal speed remains the same.
 - B. Its vertical speed increases, and its horizontal speed increases.
 - C. Its vertical speed increases, and its horizontal speed remains the same.
 - D. Its vertical speed increases and its horizontal speed decreases

8. A stand holds two white balls, A and B. At the same instant, ball A is dropped straight down, and ball B is shot straight out. Which ball will hit the ground first? (Neglect Air resistance)
- A. Ball A
 - B. Ball B
 - C. Both balls will not reach the ground.
 - D. Both balls will reach the ground at the same time.
9. Consider a plane moving with a constant speed at an elevated height above the Earth's surface. During its flight, the plane drops a package from its luggage compartment. Where will the luggage land with respect to the plane?
- A. Directly below the plane
 - B. In front of the plane
 - C. Behind the plane
 - D. None of the above
10. What factor(s) affect(s) how far a projectile will land?
- A. Angle of launch
 - B. Initial velocity
 - C. Both angle and initial velocity
 - D. None of the above