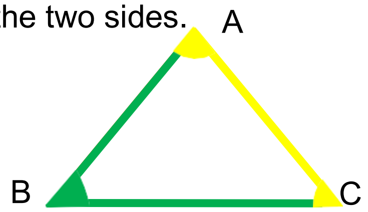
TRIANGLE CONGRUENCE

When we say included side, in a triangle, it is when the vertices of the two angles are the endpoints of the segment and that is said to be the included side of the two angles.

If we say included angle, it is when the sides of an angles are the two sides of the triangle, then the angle is said to be the included angle of the two sides.

Example, we have ΔABC**:**

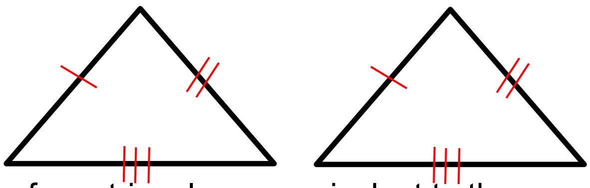


Let us identify:

Included side of $\angle A$ and $\angle C$; $\overline{AC}$

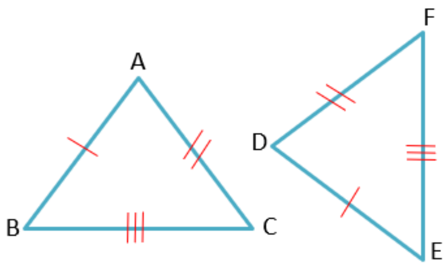
Included angle of $\overline{AB}$and $\overline{BC}$; $\angle B$

SSS (Side-Side-Side)



- If all the three sides of one triangle are equivalent to the corresponding three sides of the second triangle, then the two triangles are said to be congruent by **SSS** rule.

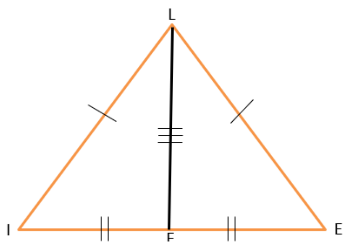
EXAMPLE:



In the given figure,

AC = DF, BC = EF and AB=DE,

hence Δ ACB ≅ Δ DFE.



$\overline{LI} \leftrightarrow \overline{LE}$ $\qquad$ $\overline{LI} \cong \overline{LE}$

$\overline{IF} \leftrightarrow \overline{EF}$ $\qquad$ $\overline{IF} \cong \overline{EF}$

$\overline{LF} \leftrightarrow \overline{LF}$ $\qquad$ $\overline{LF} \cong \overline{LF}$

Therefore, $\Delta LIF \cong \Delta LEF$ by SSS POSTULATE

SAS (Side-Angle-Side)

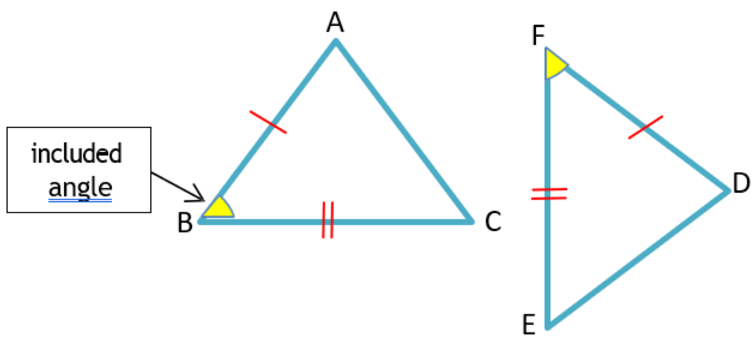
If any two sides and the included angle between the sides of one triangle are equivalent to the corresponding two sides and the angle between the sides of the second triangle, then the two triangles are said to be congruent by

SAS rule.

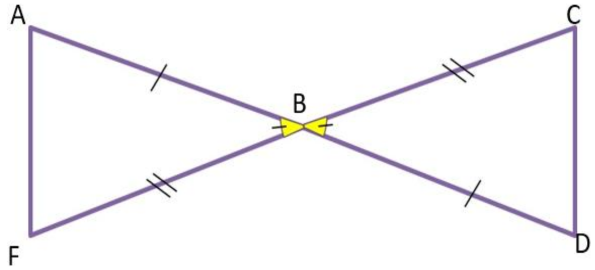
In the figure at the right,

sides AB = DF, BC = FE and angle between AB and BC equal to angle between DF and FE i.e. ∠B = ∠F.

Hence, Δ ABC ≅ Δ DEF.



EXAMPLE:

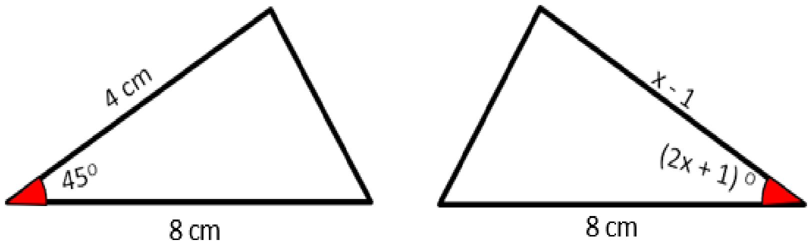


$\overline{AB} \leftrightarrow \overline{DB}$ $\overline{AB} \cong \overline{DB}$

$\angle ABF \leftrightarrow \angle CBD$ $\angle ABF \cong \angle CBD$

$\overline{FB} \leftrightarrow \overline{CB}$ $\overline{FB} \cong \overline{CB}$

Therefore, $\Delta ABF \cong \Delta DBC$ by SAS POSTULATE



$x = 22^O$

x - 1 = 4

x = 5 cm

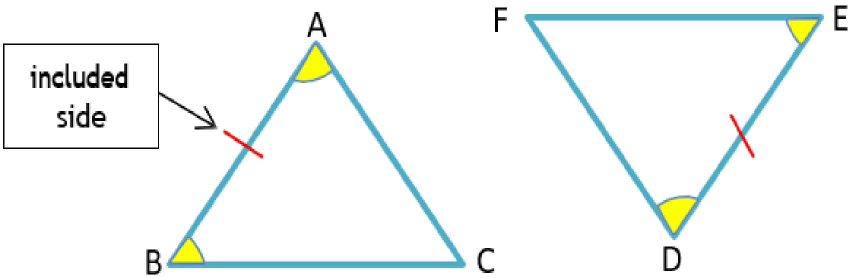
2x + 1 = 45°

2x = 44°

ASA (Angle-Side-Angle)

If any two angles and the side included between the angles of one triangle are equivalent to the corresponding two angles and side included between the angles of the second triangle, then the two triangles are said to be congruent by **ASA** rule.

EXAMPLE:



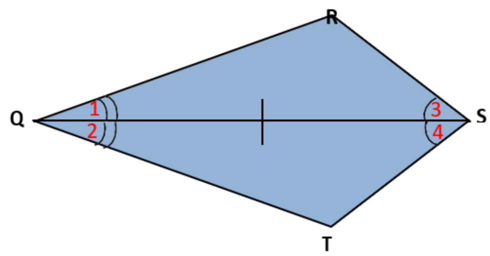
In the figure at the right, ∠ A = ∠ D, ∠ B = ∠ E and sides between ∠A and ∠B, ∠D and ∠E are equal i.e. AB= DE.

Hence, Δ ABC ≅ Δ DEF.

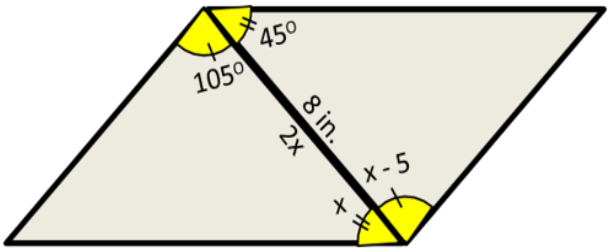
∠1 ↔ ∠2 **∠1 ≅ ∠2**

$\overline{QS} \leftrightarrow \overline{QS}$ $\overline{QS} \cong \overline{QS}$

∠3 ↔ ∠4 **∠3 ≅ ∠4**



Therefore, ΔQRS ≅ ΔQTS by ASA POSTULATE



2x = 8 in

x = 4 inches

x = 45ᴼ

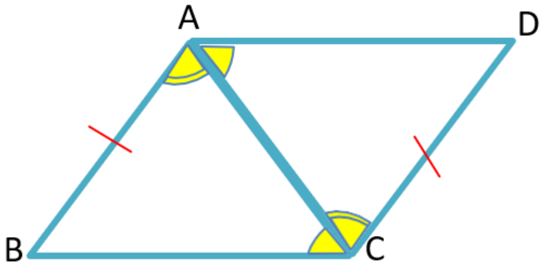
x – 5 = 105ᴼ

x = 110ᴼ

AAS (Angle- Angle-Side)

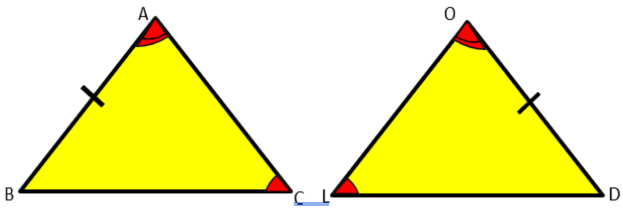
When two angles and a non-included side of a triangle are equal to the corresponding angles and sides of another triangle, then the triangles are said to be congruent.

EXAMPLE:



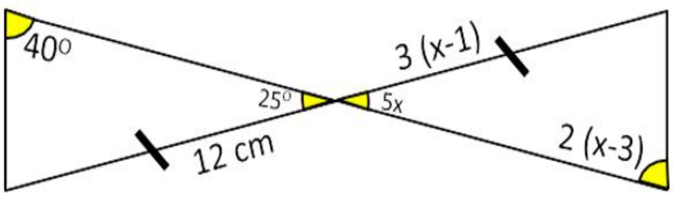
∠A ↔ ∠C	**∠A ≅ ∠C**
∠BAC ↔∠DCA	**∠BAC ≅ DCA**
$\overline{BA} \leftrightarrow \overline{CD}$	$\overline{BA} \cong \overline{CD}$

Therefore, ∆ABC ≅ ∆CDA by AAS POSTULATE



∠C ↔ ∠L	**∠C ≅ ∠L**
∠A ↔ ∠O	**∠A ≅ ∠O**
$\overline{AB} \leftrightarrow \overline{CD}$	$\overline{AB} \cong \overline{CD}$

Therefore,
∆ABC ≅ ∆OLD by AAS POSTULATE



3(x – 1) = 12 cm

3x – 3 = 12

3x = 15

x = 5 cm

.

5x = 25ᴼ

x = 5ᴼ

2(x – 3) = 40ᴼ

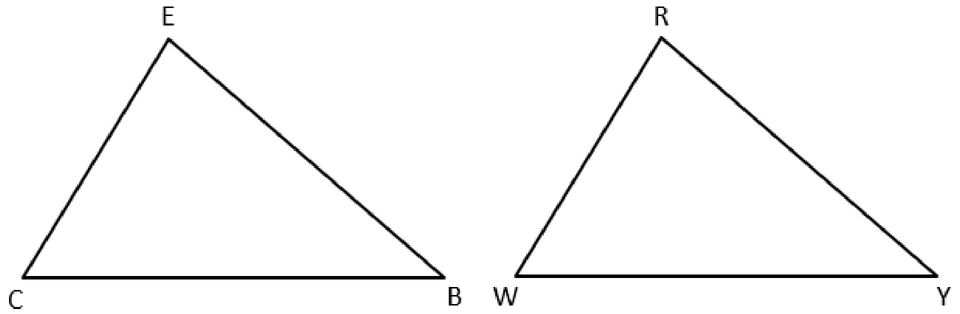
2x – 6 = 40

2x = 46

x = 23ᴼ

ACTIVITY # 1

A. State the postulate that illustrates $\Delta CEB \cong \Delta WRY$ in each case below.



1. Given: $\overline{CB} \cong \overline{WY}$; $\overline{CE} \cong \overline{WR}$; $\overline{BE} \cong \overline{YR}$
2. Given: $\angle E \cong \angle R$; $\overline{CE} \cong \overline{WR}$; $\overline{BE} \cong \overline{YR}$
3. Given: $\angle C \cong \angle W$; $\angle B \cong \angle Y$; $\overline{CB} \cong \overline{WY}$
4. Given: $\angle C \cong \angle W$; $\angle E \cong \angle R$; $\overline{BE} \cong \overline{YR}$
5. Given: $\angle B \cong \angle Y$; $\overline{EB} \cong \overline{RY}$; $\angle E \cong \angle R$
6. Given: $\overline{CB} \cong \overline{WY}$; $\overline{CE} \cong \overline{WR}$;; $\angle C \cong \angle W$

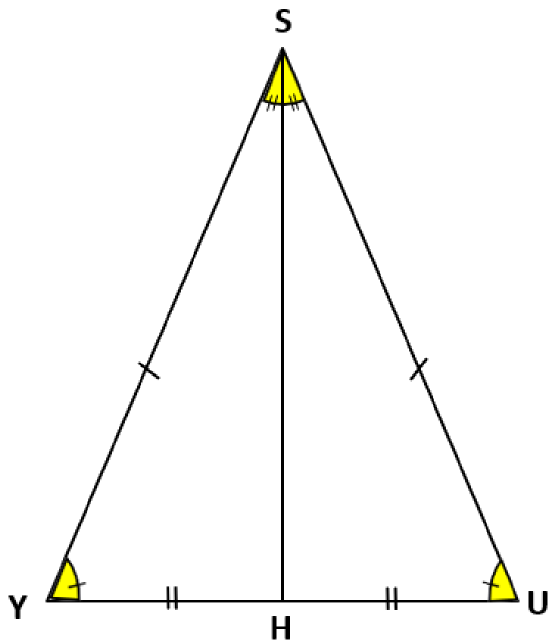
B. Given an isosceles ΔSYU, point H as the midpoint of $\overline{YU}$ and $\overline{SH}$ as angle bisector $\angle YSU$. Fill in the blank with the correct answer to illustrate that ΔYHS and ΔUHS.

1. By SSS Congruence Postulate
   $\overline{YS} \cong$ ________
   $\overline{YH} \cong$ ________
   $\overline{HS} \cong$ ________

2. By SAS Congruence Postulate
   $\overline{HS} \cong$ ___________
   ∠SHY ≅ _________
   $\overline{YH} \cong$ ___________

3. By ASA Congruence Postulate
   ∠UHS ≅ _________
   $\overline{HS} \cong$ ____________
   ∠HSU ≅ _________

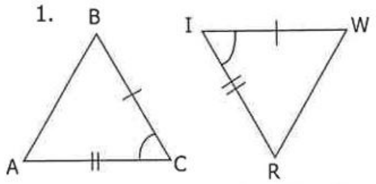
4. By AAS Congruence Postulate
   ∠U ≅ ____________
   ∠SHU ≅ _________
   $\overline{SU} \cong$ ____________



ACTIVITY # 2

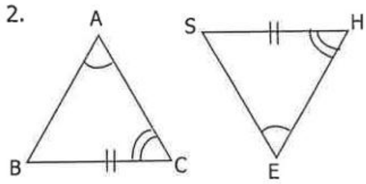
For each problem, give the correct naming order of the congruent triangles. Write the name in order on the lines for the problem number (see box at the bottom). Also, indicate which postulate or theorem is being used.

1.



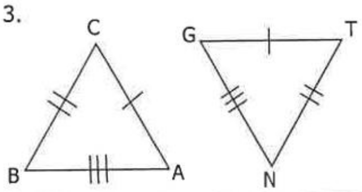
ΔABC ≅ Δ _______ by _________

2.



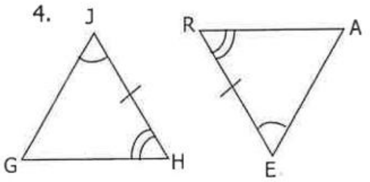
ΔABC ≅ Δ _______ by _________

3.



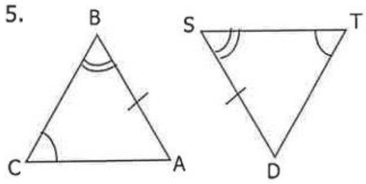
ΔABC ≅ Δ _______ by _________

4.



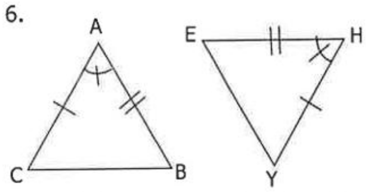
ΔGHJ ≅ Δ _______ by _________

5.



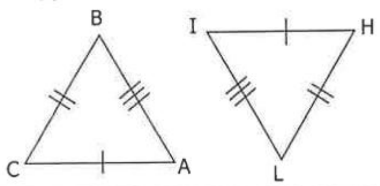
ΔABC ≅ Δ _______ by _________

6.



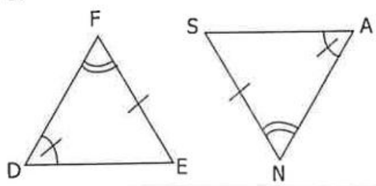
ΔABC ≅ Δ _______ by _________

7.



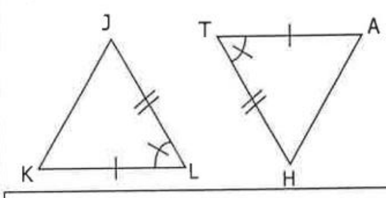
ΔABC ≅ Δ _______ by _________

8.



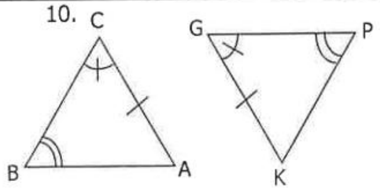
ΔDEF ≅ Δ _______ by _________

9.



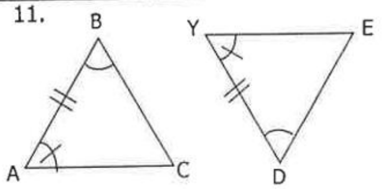
ΔJKL ≅ Δ _______ by _________

10.



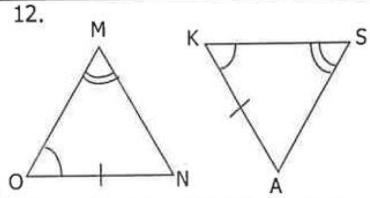
ΔABC ≅ Δ _______ by _________

11.



ΔABC ≅ Δ _______ by _________

12.



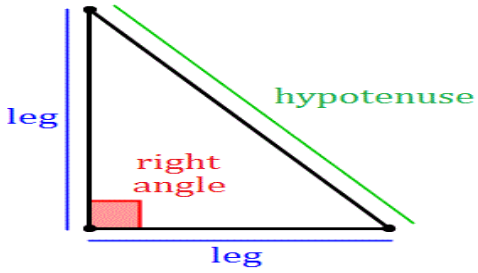
ΔMNO ≅ Δ _______ by _________

_ _ _ _ _ O _ _ N _ _ _ S _ _ E _ I _ _ _ _ _ T _
4 4 4 8 8 8 12 12 12 2 2 2 5 5 5 9 9 9 6

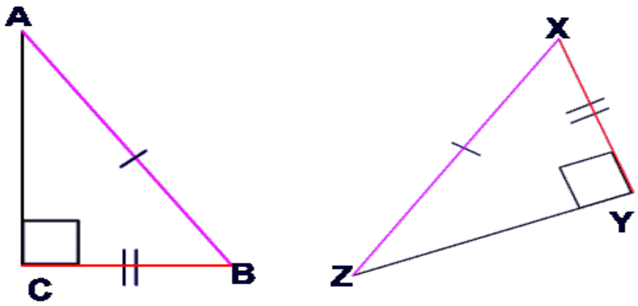
_ _ _ E E _ _ _ O _ _ N _ U _ _ _ _ T _ E _ _ I _ .
6 6 10 10 10 1 1 1 3 3 3 7 7 7 11 11 11

Knowing Right Triangles

Consider this right triangle:



Consider these two right triangles.

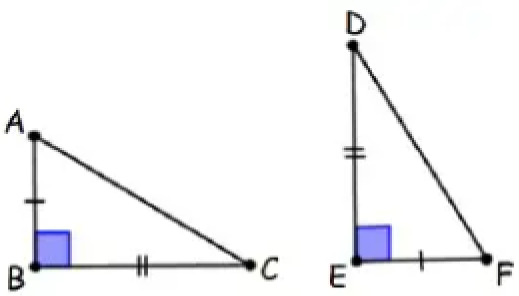


Note: If AB is congruent to XZ, BC is congruent to XY**,** are these enough to conclude that the two triangles are congruent already? Why? For right triangles, only 2 pairs of corresponding congruent parts are needed to prove that they are congruent. What are these parts? They are summarized in the following theorems:

LL CONGRUENCE THEOREM: If the legs of one right triangle are congruent to the legs of another right triangle, then the triangles are congruent.

Leg Leg Theorem :

Leg Leg (LL) Theorem is the theorem which can be used to prove the congruence of two right triangles.



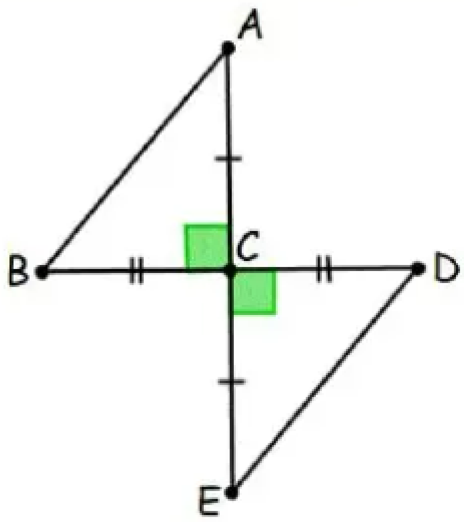
Explanation :

If the legs of one right triangle are congruent to the legs of another right triangle, then the two right triangles are congruent.

This principle is known as Leg-Leg theorem.

Leg Leg Theorem - Example

Check whether two triangles ABC and CDE are congruent.



Solution :

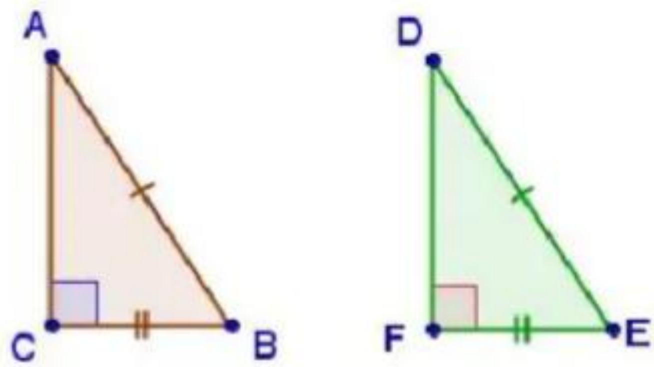
(i) Triangle ABC and triangle CDE are right triangles. Because they both have a right angle.

(i) AC = CE (Leg)

(ii) BC = CD (Leg)

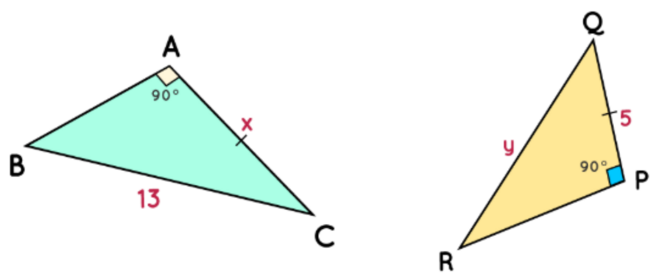
Hence, the two triangles ABC and CDE are congruent by **Leg-Leg** theorem.

The **Hypotenuse Leg Theorem** states that if the hypotenuse and one leg of a triangle are congruent to the hypotenuse and leg of another triangle, then the two triangles are congruent.



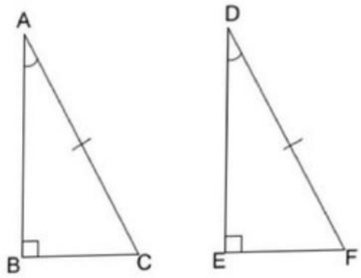
Examples:

If △ABC ≅ △PQR, what is the value of x and y?



x = 5
y = 13

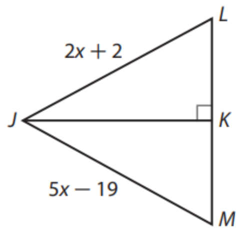
HyA CONGRUENCE THEOREM: If an acute angle and the hypotenuse of one right triangle are congruent to an acute angle and the hypotenuse of another right triangle, then the triangles are congruent.



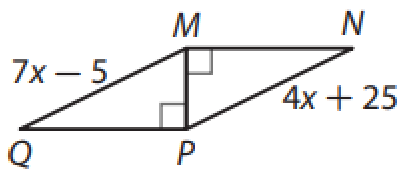
ACTIVITY # 3

What is the value of *x* that will make the given triangles congruent?

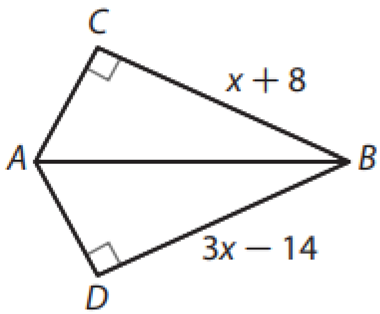
1. $\triangle JKL$ and $\triangle JKM$



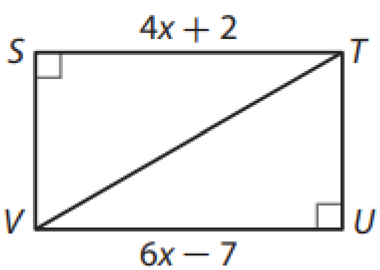
2. $\triangle MPQ$ and $\triangle PMN$



3. $\triangle ABC$ and $\triangle ABD$



4. $\triangle STV$ and $\triangle UVT$



Isosceles Triangle Theorem
If two sides of a triangle are congruent, then the angles opposite these sides are congruent.

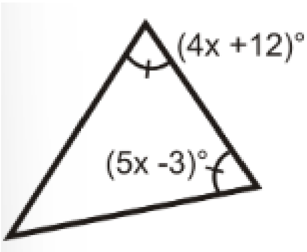
Converse of Isosceles Triangle Theorem
If two angles of a triangle are congruent, then the sides opposite these angles are congruent.

Corollary 1
A triangle is equilateral if and only if it is equiangular.

Corollary 2
Each angle of an equilateral triangle measures 60°.

Corollary 3
The bisector of the vertex angle of an isosceles triangle is perpendicular to the base at its midpoint.

Example:

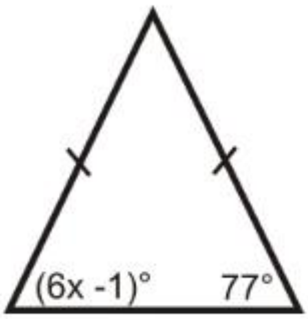


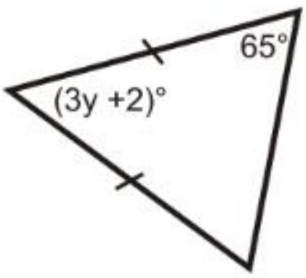
$5x - 3 = 4x + 12$
$5x - 4x = 12 + 3$
$x = 15$

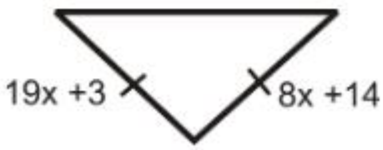
Therefore, the base angles are 72^{O} and the vertex angle is 36^{O}

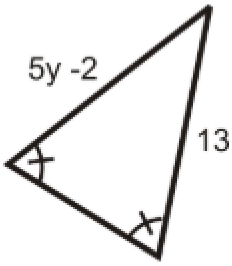
ACTIVITY # 4

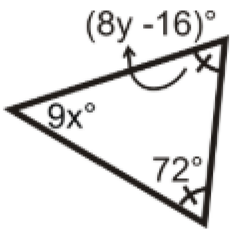
Solve for the value of the missing variable, given by an isosceles triangle.

1.


2.


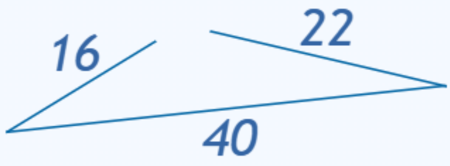
3.


4.


5.


Triangle Inequality
The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

- If a side is **longer** than the other two sides there is a gap:



- If a side is **equal** to the other two sides it is not a triangle (just a straight line back and forth).

When the three sides are **a**, **b** and **c**, we can write:

- a < b + c
- b < a + c
- c < a + b

NOTE:

- If two sides of a triangle are not congruent, then the larger angle lies opposite the longer side.
- If two angles of a triangle are not congruent, then the longer side lies opposite the larger angle.

ACTIVITY # 5

Determine if the sets of lengths below can make a triangle. If not, state why.

1. 6, 6, 13
2. 1, 2, 3
3. 7, 8, 10
4. 5, 4, 3
5. 23, 56, 85
6. 30, 40, 50
7. 7, 8, 14
8. 7, 8, 15